Exercise 7.1
Use the symmetric molar regular free energy of mixing for a binary alloy of $A$ and $B$ at fixed pressure.

$$\Delta G^{RS} = \Delta H^{RS} - T\Delta S^{RS}$$

$$= X_A X_B \omega^{RS} - T[(-R)(X_A \log X_A + X_B \log X_B)]$$

(1)

and do the following.

7-1-i Determine the critical temperature $T_c$ in terms of $\Omega^{RS}$ and $R$.

7-1-ii Plot the equilibrium compositions from $T = 0$ to $T = 1.1 T_c$.

7-1-iii Plot $\mu_A(X_B)$ for $0 \leq X_B \leq 1$.

7-1-iv Plot $\mu_A(X_B)$ versus $\mu_B(X_B)$ for $0 \leq X_B \leq 1$.

Exercise 7.2
Consider the stagnation problem associated with the disappearance of a nearly cylindrical grain in a thin sheet with thickness $h$.

Figure 7-2-i: Illustration of disappearing grain in a thin sheet. The circular boundary groove, radius $R_g$, which forms on each surface creates a pinning force resisting boundary motion.
If a groove develops as shown, the grain boundary can become “pinned.”

7-2-i Show that, for a pinned boundary, \( r(z) = R_w \cosh(z/R_w) \) is the equilibrium shape of the grain boundary if all interfaces are isotropic.

7-2-ii Calculate the net force on the groove due to the grain when the radius of the groove \( R_y = h \).
   Note that \( \alpha \beta = \cosh(\beta / 2) \) has two solutions when \( \alpha = 1 \), \( \beta = 1.1787 \) and \( \beta = 4.2536 \).

7-2-iii \( \alpha \beta = \cosh(\beta / 2) \) ceases to have any solutions when \( \alpha < 0.75 \). What happens to the grain when \( R_y \) deceases to about \( 3/4\) \( h \)?

**Exercise 7.3**
Calculating the fastest growing and smallest unstable wavelengths for a cylinder which is evolving due to surface diffusion.

Start with a uniform cylinder and perturb with an infinitessimal pertubation \( R(z,t) = R_o + \epsilon(t) \sin 2\pi z / \lambda \). Use the small slope approximation for the surface diffusion equation:

\[
\frac{\partial R}{\partial t} = D_s \frac{\partial^2 \kappa}{\partial z^2}
\]

Find an expression for \( \epsilon(t) \) and maximize with respect to \( \lambda \).

**Exercise 7.4**
Determine the fastest growing and smallest unstable wavelengths (if they exist) for:

7-4-i a nonconserved order parameter, \( \eta(x) \) with homogeneous free energy density:

\[
f(\eta) = f_s((1 + \eta)(1 - \eta))^4
\]

7-4-ii a conserved order parameter, \( c(x) \) with homogeneous free energy density:

\[
f(c) = \frac{2^8 f_s}{c_\beta - c_\alpha}((c - c_\alpha)(c - c_\beta))^4
\]