1 Voronoi Tessellation on a Curvy Beast

1.1 Introduction

Every physical object has a surface; many have internal interfaces. The interaction of an object with its environment is mediated by—and frequently defined by–the surface's material properties. For composite objects, interfaces mediate the response of a component part with its immediate neighborhood. Thus, any *complete* account of the environmental behavior an object must include the analysis of its surface properties and geometry. Such surface and interface property analyses are frequently in the province of Materials Science and Surface Physics [**?**]. However, as in any discipline, analyses are limited to problems of the disciplines direct and derivative interests: materials scientists focus on objects either as nature presents them, or to the extent that natural processes effect the evolution of a material structure.

A holistic perspective of surface properties requires interdisciplinary cooperation, or *dialogues* between disciplines with collective interests on surface behavior.

Architectural Design necessarily includes the appearance and function of surfaces (i.e., façades). The role of surfaces in design-related sciences extends beyond the property-studies of Surface Physics; instead, a designer has an opportunity to create a novel object and its surfaces. Whatever the goals of the designer, the object—whether a physical realization or concept—cannot be independent of its surface's physical properties. Design without regard to materials properties is not a holistic approach. Design and properties lie at the interface of composite disciplines.

Sub-disciplines of mathematics, such as differential and computational geometry, focus on surfaces as abstract objects. The mathematical vocabulary of surface abstraction creates a productive milieu for interdisciplinary dialogues between designers and materials scientists at the very least.

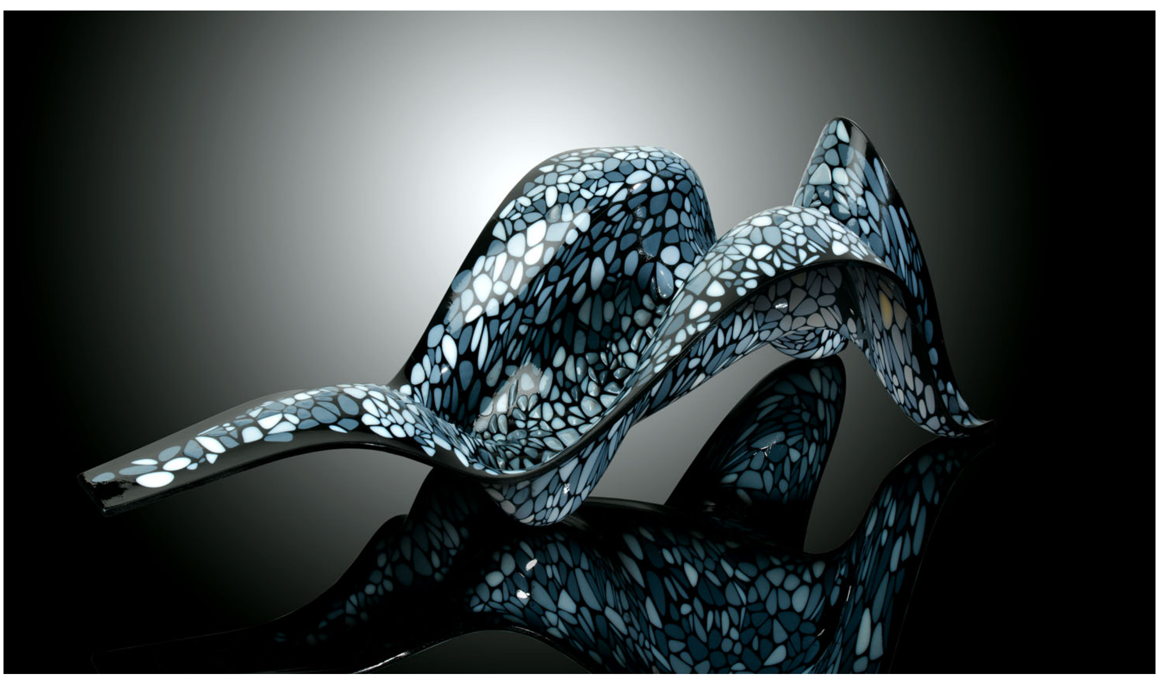

Figure 1: Photo of Beast. Description and credits here.

In its rudimentary form, such a dialogue may begin with designer's creation of a NURBS surface that is consigned to materials science for material choices or property calculations. The materials properties are projected onto the NURBS surface and transmitted to the designer; this dialogues iterates until a solution—or resolution—appears. Such simple approaches evolve to a composite approach. We believe the seeds of a composite discipline can be sown such a milieu.

To make the above abstract discussion concrete, we describe a particular case below: the design, creation, and production of a materials-informed texture for the chaise lounge shown in Fig. 1.

Architectural textures often—and historically–derive from tilings. While the design of such tilings can be wonderfully complex and beautiful, it is likely most common that a designer considers the color and other visible characteristics of tile properties. However, examples of further considerations—such as variable hardness, tactility, and acoustic response—do exist. We consider deeper connections between properties and tilings.

1.2 Voronoi Textures on Curved Surfaces

Generically, a tiling (or tessellation) is surface-covering composed of separable components, or tiles. The tiles can be chosen from a set of one or more types. Simple examples include square and hexagonal tilings. More complex examples include Penrose tilings, randomly colored uniform-polygon tiles, or the hexagons and pentagons that compose a Buckminster sphere.

A Voronoi tessellation is an example of a tiling that is generated by a random point-process; that is, the tiling is develops algorithmically (to be described below) from points that appear on a surface by *some* random process; in some disciplines, the generated set of random points are called point clouds. In most cases, the points are generated from a homogeneous, uniform, random distribution (also called a Poisson process). In such a uniform process, no position is favored over another: the Voronoi tiling is segmented object at the length scale of the average distance between points; but, the tiling is homogeneous when averaged over larger distances. However, the Voronoi tessellation need not derive from such a uniform random process. The object in Fig. 1 is an example: the individual texture components (i.e., the splined loops) derive from a Voronoi tessellation, but the point cloud density is a function (in the mathematical sense of function) of the chair's local geometric curvature. Thus, because local stiffness, tactility, and appearance depend on local tile size, the object's geometry is intrinsically coupled to material behavior—and as we shall explain the converse is also true.

Voronoi tilings appear in disparate fields (?an example list here?). Each tile is defined by the set of points that lie closest to each generated point (i.e., a hexagonal tiling derives from a hexagonal lattice of generating points). Many algorithms exist that produce simple versions of Voronoi tessellations from point clouds. Fast Voronoi algorithms have been developed with computational geometry techniques, but the computations are generally time-consuming.

Because the Voronoi definition includes a *distance function* (i.e., "closeness" is a comparison of distances), the Voronoi construction depends on what is meant by distance. In the simple case of a Voronoi construction on a two-dimensional plane, the common choice is the Euclidean distance ($d = \sqrt{x^2 + y^2}$, also called the $L2$-norm), but there are an infinite number of ways to define a distance. For the Euclidean distance, the Voronoi tiles are all polygons for which each shared polygon-edge is (a segment of) the perpendicular bisector of the ray joining the the generating-points centers of the two tiles. (The set of all rays connecting neighboring Voronoi centers is a skeletal structure which is "dual" to the Voronoi construction, and called a *Delaunay triangulation.*) However, the resulting polygonal structure is particular to the definition of distance.

On curved surfaces, such as the uniformly curved sphere, the definition of distance is more complicated and the tile edges have out-of-surface curvature. For non-uniformly curved objects, such as that in Fig. 1, the distance definition is more complicated yet—and the algorithms to find their corresponding tessellation are complicated and typically undeveloped. The tile edges have non-uniform in-plane and out-of-plane curvatures.

Paragraph here about distance metrics in curved space.

There are methods to use a simplified distance metric to produce a Voronoi tessellation on a curved surface. An example follows. A point cloud can be generated on the vertical projection of a surface, or a point cloud on a curved surface can be projected vertically to a plane. In the first case, a uniform cloud distribution on the plane produces a non-uniform point cloud on the surface; in the second case, a uniform point cloud on the surface produces a non-uniform cloud on the plane. In either case, the Euclidean algorithm can be used to produce a Voronoi tessellation on the projected plane, and that tessellation can be projected back onto the surface. Such methods are possible on limited surface types (i.e., *graphs* of the form $z = f(x, y)$). The surface's angle of inclination from the vertical produces the non-uniformity of the point cloud: points become arbitrarily close in regions where the surface approaches verticallity.

The choice of algorithm can have esthetic and property-related consequences. The effects of this algorithm are illustrated in Fig. 2.

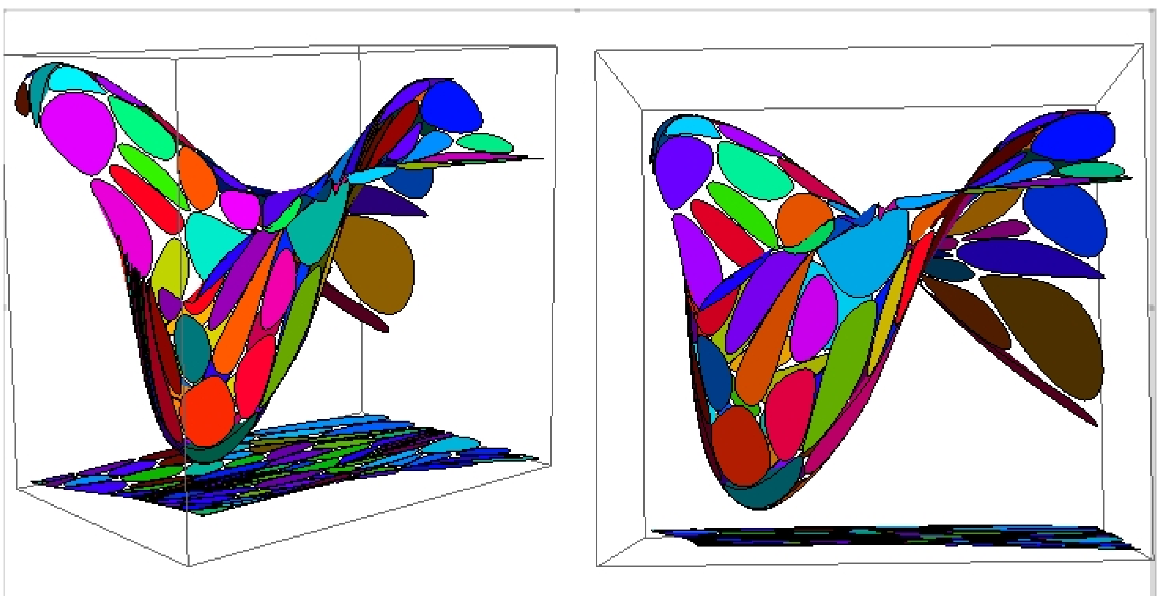

Figure 2: Example of Voronoi construction with a vertical projection. The tile-squeezing correlates to angle between the surface normal and the projection vector. Not a good figure, will generate another

A full implementation of an $L2$-norm on $u{-}v$ embedded in a surface of the form $\{x(u, v), y(u, v), z(u,$ would computationally prohibitive. The object in Fig. 1 was produced with an algorithm that approximates this norm.

A paragraph or two, kind of technical, describing the algorithm and its shortcomings

1.3 Vertical Weighted Material Choice

1.4 Tessellation to Spline Loops

1.5 Sectioning Model for Production

1.6 Numerical Model/Data Transmission to Manufacturer