3.016 Fall 2003 Planning

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1 Background Statement

1.1 Purpose

The purpose of this course is to give undergraduates in materials science and engineering an introduction to most of the essential mathematical concepts that they will need as part of a complete undergraduate education. This course is intended for sophomore who are just beginning their major in materials science and engineering. Students are expected to understand and be able to apply concepts from basic calculus (integration and differentiation of functions of a single variable, limits and continuity, fundamental theorem of calculus, Taylor series) and from multi-variable calculus (partial differentiation, vector differential operators, multidimensional integration, vector algebra) although some concepts will be reviewed as needed.

1.2 Course Content

The course is taught in conjunction with a MIT course (3.012) on symmetry, structure, energetics and thermodynamics—many of the pedagogical examples of mathematical application will derive from this course. Additionally, this Maths for MSE course must serve as the background for much of the major and therefore the topics are necessarily plentiful and disparate. Just how to compress the mathematical material for a major into a single 8 (*3-0-5*) unit course[1] requires subjectivity and compromise on the part of the instructor in charge. My deliberations and choices were frequented with conflicting themes of rigor/depth and familiarity/breadth. When a choice was necessary, one of breadth over depth was made—I reason that it is possible for a student to go back for depth, but impossible to use an unfamiliar mathematical technique when needful. Nevertheless, the list of topics below probably still do not suffice as a *complete* set of mathematical topics. However, the list is the best one that I could construct with curricular constraints.

1.3 Syllabus/Order/Textbook

Another choice is the arrangement of topics and amount of time dedicated to each topic. Here, the conflict is between that of logical precedence of topics and the order of material taught

[1] Units at MIT are assigned under the following schema: *Lec-Lab-Out* where *Lec* is the number of lecture/recitation hours, *Lab* is the number of laboratory hours, and *Out* is the number of outside (reading, preparation, homework) hours per week. One MIT unit represents about 14 hours of work on the average.

in the other course (3.012). I cannot ascertain if the correct balance has been achieved—this should be revisited after the course is over.

It has been my experience that sophomores prefer the structure that is provided by a textbook. Thus, I have chosen a fairly general textbook on applied mathematics (E. Kreyszig, *Advanced Engineering Mathematics*, eighth ed., J.W. Wiley, $\approx$ 1200 pages) and have rearranged the topic order within the book to fit the needs of course. Extra material pertaining to materials science specifically will be created and placed on the web.

I have identified 66 sections of the book (330 pages in total) as required reading. Spreading the over 22 lectures is 15 pages of reading time per lecture which I would conservatively estimate as one hour of outside reading per lecture. There are 27 lectures in total, about 5 lectures will utilize outside material. After outside reading, this leaves about 2-3 hours a week for homework.

1.4 Software Choice: *Mathematica*®

The practice of and the pedagogy of Applied mathematics is significantly aided by the existence of computer symbolic algebra/calculus packages and visualization. I conclude that an introduction and some proficiency training in one of these tools is pedagogically sound. There are a number of different good choices including *MatLab*, *Maple*, *MSOffice Products*, and *Mathematica*. I've chosen *Mathematica* as a vehicle for this course; in my judgment, it is the most general and powerful mathematical software—although it is perhaps too comprehensive to be mastered rapidly. *Mathematica*® is freely available to all MIT students. This particular choice should be revisited after the course.

The first five lectures will introduce students to *Mathematica*® usage, visualization, and programming. Throughout the course, examples will be presented as worked problems in *Mathematica*® and students will turn in their homework as either *Mathematica*® printouts or electronic notebooks. Much of this first five lectures will resemble the 16-hour Wolfram training course that I took recently.

1.5 Grading, Homework, and Exams

For a materials scientist and engineer, mathematics should be a readily available tool for exploration and a language to compactly describe concepts. It is particularly useful in materials science which serves as a bridge between so many physical and engineering sciences—just as mathematics does.

To encourage mathematics as a tool for exploration, I have decided to emphasize worked problem sets over exams. A student's entire grade will be based on homework—there will be no exams. There is a potential downside to the no-exam concept: that students will not have an studying opportunity to tie material together. It is my hope that, because students will be using this material in 3.012 and because I can make the homework a more relevant and difficult, the advantages of no-exams will prevail.

I also think that student cooperation on homework 1) is a useful way for students to work and learn together; 2) teaches professional skills; 3) creates camaraderie between students within the same major; 3) is inevitable. There is a built-in conflict between student homework cooperation and a just method of assigning a final grade.

I have decided to make each assigned homework have two parts.

individual There will be an individual part that each student will do their own work—these will include simple problems from the textbook.

group There will be a group part of each homework assignment. Students will be randomly assigned[2] to groups of size 5 or 6 and each group will turn in one solution for the entire group.

As part of the individual task, each student in the group will be required to turn in their own evaluation of the distribution of effort on the group section.

2 Course Topics and Syllabus

2.1 Week of 1-5 September

	Topics	**Reading**
M 09/01	Registration Day, No Lectures	
W 09/03 Lect. 1	Course Organization and Introduction to Mathematica	Mathematica Notes *I*
F 09/05 Lect. 2	Introduction to Mathematica, assignment and evaluation, rules and replacement, procedural and functional programming	Mathematica Notes *II*

2.2 Week of 8-12 September

	Topics	**Reading**
M 09/08 Lect. 3	Mathematica Graphics: basic plotting, data, two and three dimensional plotting, graphics primitives, formatting.	Mathematica Notes *III*
W 09/10 Lect. 4	Mathematica: symbolic and numeric calculations, linear algebra, roots of equations	Mathematica Notes *IV*
F 09/12 Lect. 5	Mathematica: Functional programming, packages, and file input/output	Mathematica Notes *V*

[2]New random assignments performed for each problem set

2.3 Week of 15-19 September

	Topics	**Reading**
M 09/22 Lect. 6	Linear Algebra: matrix operations, interpretations of matrix operations, multiplication, transposes, index notation.	*Kreyszig* **6.1, 6.2, 6.3, 6.4** (pages: 304–309, 312–318, 321–323, 331–336)
W 09/17 Lect. 7	Linear algebra: solutions to linear systems of equations, determinants, matrix inverses, linear transformations and vector spaces	*Kreyszig* **6.5, 6.6, 6.7, 6.8** (pages: 338–341, 341–347, 350–357, 358–364)
F 09/19 Lect. 8	Complex numbers: complex plane, addition and multiplication, complex conjugates, polar form of complex numbers, powers and roots,r exponentiation, hyperbolic and trigonometric forms	*Kreyszig* **12.1, 12.2, 12.6, 12.7** (pages: 652–656, 657–662, 679–682, 682–685)

2.4 Week of 22-26 September

	Topics	**Reading**
M 09/22	Holiday, No Lectures	
W 09/24 Lect. 9	Matrix Eigenvalues: eigenvalue/eigenvector definitions, invariants, principal directions and values, symmetric, skew-symmetric, and orthogonal systems, orthogonal transformations.	*Kreyszig* **7.1, 7.2, 7.3** (pages: 371–375, 376–379, 381–384)
F 09/26 Lect. 10	Hermitian Forms, Similar matrices, eigenvalue basis, diagonal forms	*Kreyszig* **7.4, 7.5** (pages: 385–389, 392–396)

2.5 Week of 29 Sept–3 Oct.

Laboratory Week: 3.016 does not meet.

2.6 Week of 6-10 October

	Topics	**Reading**
M 10/06 Lect. 11	Vector Calculus: vector algebra, inner products, cross products, determinants as triple products, derivatives of vectors	*Kreyszig* **8.1, 8.2, 8.3, 8.4** (pages: 401–406, 408–413, 414–421, 423–427)
W 10/08 Lect. 12	Multi-variable Calculus: Curves and arc length, differentials of scalar functions of vector arguments, chain rules for several variables, change of variable and thermodynamic notation, gradients and directional derivatives	*Kreyszig* **8.5, 8.8, 8.9** (pages: 428–433, 444–446, 446–452)
F 10/10 Lect. 13	Vector Differential Operations: divergence and its interpretation, curl and its interpretation.	*Kreyszig* **8.10, 8.11** (pages: 453–456, 457–459)

2.7 Week of 13-17 October

	Topics	**Reading**
M 10/13	Holiday, No Lectures	
W 10/15 Lect. 14	Path Integration: integral over a curve, change of variables, multidimensional integrals.	*Kreyszig* **9.1, 9.2, 9.3** (pages: 464–470, 471–477, 478–484)
F 09/26 Lect. 15	Multidimensional Forms of the Fundamental theorem of calculus: Green's theorem in the plane, surface representations and integrals,	*Kreyszig* **9.4, 9.5, 9.6, 9.7** (pages: 485–490, 491–495, 496–505, 505-509)

2.8 Week of 20–24 October

Laboratory Week: 3.016 does not meet.

2.9 Week of 27-31 October

	Topics	**Reading**
M 10/27 Lect 16	Multi-variable Calculus: triple integrals and divergence theorem, applications and interpretation of the divergence theorem, Stoke's theorem.	*Kreyszig* **9.8, 9.9** (pages: 510–514, 515–520)
W 10/29 Lect. 17	Periodic functions: Fourier series, Interpretation of Fourier coefficients, convergence, odd and even expansions	*Kreyszig* **10.1, 10.2, 10.3, 10.4** (pages: 527–528, 529–536, 537–540, 541–546)
F 10/31 Lect. 18	Fourier Theory: Complex form of Fourier Series, Fourier Integrals, Fourier Cosine and Sine transforms, The Fourier transforms	*Kreyszig* **10.5, 10.8, 10.9, 10.10** (pages: 547–549, 557–563, 564–568, 569-575)

2.10 Week of 3-7 November

	Topics	**Reading**
M 11/03 Lect 19	Ordinary Differential Equations: physical interpretations, geometrical interpretations, separable equations	*Kreyszig* **1.1, 1.2, 1.3** (pages: 2–8, 10–12, 14–18)
W 11/05 Lect. 20	ODEs: Derivations for simple models, exact equations and integrating factors, the Bernoulli equation	*Kreyszig* **1.4, 1.5, 1.6** (pages: 19–22, 25–31, 33–38)
F 11/07 Lect. 21	Higher Order Differential Equations: homogeneous second order, initial value problems, second order with constant coefficients, solution behavior	*Kreyszig* **2.1, 2.2, 2.3** (pages: 54–70, 72–75, 76–80)

2.11 Week of 10–14 November

Laboratory Week: 3.016 does not meet.

2.12 Week of 17-21 November

	Topics	**Reading**
M 11/17	Laboratory Continues, No Lecture	
W 11/19 Lect. 22	Differential operators, damped and forced harmonic oscillators, nonhomogeneous equations.	*Kreyszig* **2.4, 2.5, 2.8** (pages: 81–83, 83–89, 101–103)
F 11/21 Lect. 23	Resonance phenomena, higher order equations, beam theory.	*Kreyszig* **2.11, 2.13(beam theory only)** (pages: 111–116, 130–131)

2.13 Week of 24-28 November

	Topics	Reading
M 11/24 Lect. 24	Systems of differential equations, linearization, stable points, classification of stable points.	*Kreyszig* **3.1, 3.2** (pages: 152–157, 159–161)
W 11/26 Lect. 25	Linear Differential equations: phase plane analysis and visualization	*Kreyszig* **3.3, 3.4** (pages: 161–169, 170–174)
F 11/28	Holiday, No Lecture	

2.14 Week of 1-5 December

	Topics	Reading
M 12/01 Lect. 26	Solutions to Differential Equations: Legendre's equation, orthogonality of Legendre polynomials, Bessel's equation and Bessel functions	*Kreyszig* **4.3, 4.5, 4.6** (pages: 111–116, 130–131, 228–232)
W 12/03 Lect. 27	Sturm-Louiville Problems: eigenfunction, orthogonal functional series, eigenfunction expansions	*Kreyszig* **4.7, 4.8** (pages: 233–238, 240–248)
F 12/05	Laboratory continues, No more Maths Lectures	

2.15 Week of 8–12 December

Laboratory Week: 3.016 does not meet.

3 Homework Schedule

Some homeworks will be longer than others and therefore the weights will not be equal.

Homework	**weight factor**	**Handed Out**		**Due Back**	
		Lecture	Date	Lecture	Date
1	0.75	Lect. 1	3 Sept.	Lect. 5	12 Sept.
2	1.0	Lect. 4	10 Sept.	Lect. 8	19 Sept.
3	1.25	Lect. 7	17 Sept.	Lect. 10	26 Sept.
4	2.0	Lect. 11	6 Oct.	Lect. 15	17 Oct.
5	1.0	Lect. 16	27 Oct.	Lect. 19	3 Nov.
6	1.0	Lect. 18	31 Oct.	Lect. 21	7 Nov.
7	1.0	Lect. 22	19 Nov.	Lect. 25	26 Nov.
8	0.75	Lect. 24	24 Nov.	Lect. 27	3 Dec.