3.016 Problem Set #1, 2010

I1-1

Define the function fOfX, and let *Mathematica* express it

```
fOfX = 1 / (1 / 2 + Sin[x] ^ x)
```

$$\frac{1}{\frac{1}{2} + \text{Sin}[x]^x}$$

Take derivative of the function fOfX, and let *Mathematica* simplify the result

```
Simplify[D[fOfX, x]]
```

$$-\frac{4\ (x\ \text{Cot}[x] + \text{Log}[\text{Sin}[x]])\ \text{Sin}[x]^x}{(1 + 2\ \text{Sin}[x]^x)^2}$$

I1-2

2.1

Assign x = (1+y)

```
x = (1 + y);
```

Expand x^2 and x^4

```
Expand[(1 + y) ^ 2]
Expand[(1 + y) ^ 4]
```

$1 + 2\ y + y^2$

$1 + 4\ y + 6\ y^2 + 4\ y^3 + y^4$

2.2

Define the variable z

```
z = Exp[Integrate[(s / (1 + s ^ 2)), {s, 0, x ^ 2}, Assumptions → y ∈ Reals]]
```

$$\sqrt{2 + y\ (2 + y)\ (2 + y\ (2 + y))}$$

Try simplifying the result

```
Simplify[z, Assumptions → y ∈ Reals]
```

$\sqrt{2 + y\ (2 + y)\ (2 + y\ (2 + y))}$

Factor the result

```
Factor[z]
```

$\sqrt{2 + 4\ y + 6\ y^2 + 4\ y^3 + y^4}$

Verify that $\sqrt{1 + (1 + y)^4}$ is equivalent to $\sqrt{2 + 4\ y + 6\ y^2 + 4\ y^3 + y^4}$

```
Expand[1 + (1 + y)^4]
```

$2 + 4\ y + 6\ y^2 + 4\ y^3 + y^4$

```
Sqrt[1 + (1 + y)^4]
```

$\sqrt{1 + (1 + y)^4}$

- **2.3**

Take the derivative as required, result is assigned to dThing

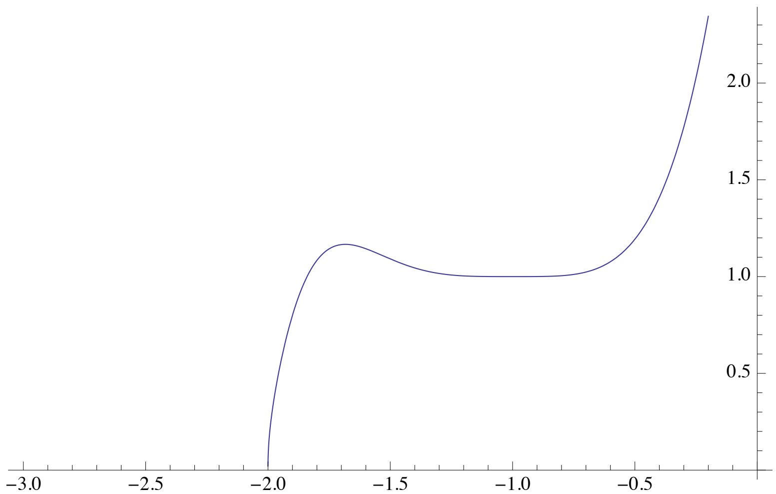
```
dThing = D[(1 + x)^z, y]
```

$$(2 + y)^{\sqrt{2+y\ (2+y)\ (2+y\ (2+y))}} \left(\frac{\sqrt{2 + y\ (2 + y)\ (2 + y\ (2 + y))}}{2 + y} + \frac{(y\ (2 + y)\ (2 + 2\ y) + y\ (2 + y\ (2 + y)) + (2 + y)\ (2 + y\ (2 + y)))\ \mathrm{Log}[2 + y]}{2\ \sqrt{2 + y\ (2 + y)\ (2 + y\ (2 + y))}}\right)$$

- **2.4**

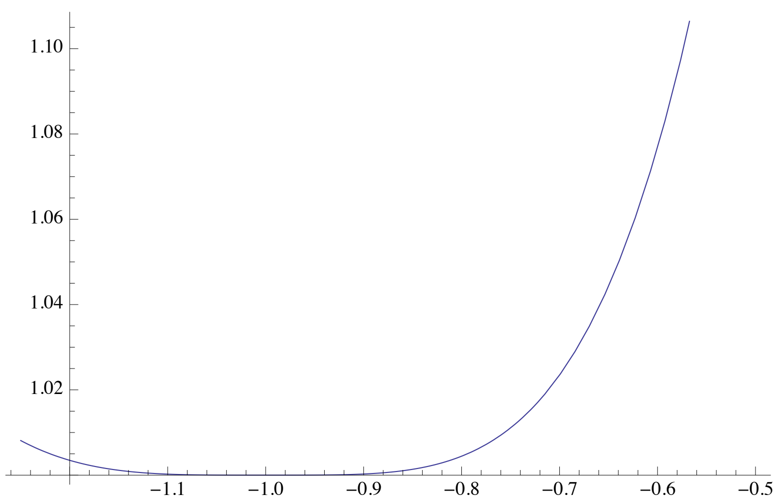
Plot the above derivative dThing within -3 < y < 0

```
Plot[dThing, {y, -3, 0}]
```



- **Interesting very flat minimum between -1.5 and -0.5, explore nearby functional behavior...**

```
Plot[dThing, {y, -1.25, -0.5}]
```




```
FindMinimum[dThing, {y, -1}]
```

FindMinimum::fmgz :
Encountered a gradient that is effectively zero. The result returned may not be a minimum; it may be a maximum or a saddle point. ≫

```
{1., {y → -1.}}
```

Must be a very flat miniume. See if we can verify the minimum at y = -1.

```
D[dThing, y] /. y → -1
```

0

```
dThing /. y → -1
```

1

I1-3

3.1

Define aFunc that returns a list with variables x and n,

```
aFunc[x_, n_] := Table[x^i, {i, 0, n}]
```

Obtain the list using aFunc with x = 2 and n = 12,

```
aFunc[2, 12]
```

{1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096}

3.2

Devide every member of the list by 2,

```
aFunc[2, 12] / 2
```

$\left\{\frac{1}{2}, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048\right\}$

The above is a list of 2^i for i = -1, 0, 1, 2, ..., n-2. Now doing the substraction,

```
aFunc[2, 12] - aFunc[2, 12] / 2
```

$\left\{\frac{1}{2}, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048\right\}$

This makes sense because each member is 2^i - 2^(i-1) = (1-1/2) 2^i = 2^(i-1).

3.3

Simply evaluate aFunc with x = Carter and n = 100 will give the desired list,

```
aFunc[Carter, 100]
```

$\{1, \text{Carter}, \text{Carter}^{2}, \text{Carter}^{3}, \text{Carter}^{4}, \text{Carter}^{5}, \text{Carter}^{6}, \text{Carter}^{7}, \text{Carter}^{8}, \text{Carter}^{9}, \text{Carter}^{10}, \text{Carter}^{11}, \text{Carter}^{12}, \text{Carter}^{13}, \text{Carter}^{14}, \text{Carter}^{15}, \text{Carter}^{16}, \text{Carter}^{17}, \text{Carter}^{18}, \text{Carter}^{19}, \text{Carter}^{20}, \text{Carter}^{21}, \text{Carter}^{22}, \text{Carter}^{23}, \text{Carter}^{24}, \text{Carter}^{25}, \text{Carter}^{26}, \text{Carter}^{27}, \text{Carter}^{28}, \text{Carter}^{29}, \text{Carter}^{30}, \text{Carter}^{31}, \text{Carter}^{32}, \text{Carter}^{33}, \text{Carter}^{34}, \text{Carter}^{35}, \text{Carter}^{36}, \text{Carter}^{37}, \text{Carter}^{38}, \text{Carter}^{39}, \text{Carter}^{40}, \text{Carter}^{41}, \text{Carter}^{42}, \text{Carter}^{43}, \text{Carter}^{44}, \text{Carter}^{45}, \text{Carter}^{46}, \text{Carter}^{47}, \text{Carter}^{48}, \text{Carter}^{49}, \text{Carter}^{50}, \text{Carter}^{51}, \text{Carter}^{52}, \text{Carter}^{53}, \text{Carter}^{54}, \text{Carter}^{55}, \text{Carter}^{56}, \text{Carter}^{57}, \text{Carter}^{58}, \text{Carter}^{59}, \text{Carter}^{60}, \text{Carter}^{61}, \text{Carter}^{62}, \text{Carter}^{63}, \text{Carter}^{64}, \text{Carter}^{65}, \text{Carter}^{66}, \text{Carter}^{67}, \text{Carter}^{68}, \text{Carter}^{69}, \text{Carter}^{70}, \text{Carter}^{71}, \text{Carter}^{72}, \text{Carter}^{73}, \text{Carter}^{74}, \text{Carter}^{75}, \text{Carter}^{76}, \text{Carter}^{77}, \text{Carter}^{78}, \text{Carter}^{79}, \text{Carter}^{80}, \text{Carter}^{81}, \text{Carter}^{82}, \text{Carter}^{83}, \text{Carter}^{84}, \text{Carter}^{85}, \text{Carter}^{86}, \text{Carter}^{87}, \text{Carter}^{88}, \text{Carter}^{89}, \text{Carter}^{90}, \text{Carter}^{91}, \text{Carter}^{92}, \text{Carter}^{93}, \text{Carter}^{94}, \text{Carter}^{95}, \text{Carter}^{96}, \text{Carter}^{97}, \text{Carter}^{98}, \text{Carter}^{99}, \text{Carter}^{100}\}$

I1-4

■ 4.1

Evaluate the indefinite integral as required,

```
indefInt = (2 Sqrt[alpha] / Pi) Integrate[Exp[-alpha s^2], s]
```

$$\frac{\mathrm{Erf}\left[\sqrt{\mathrm{alpha}}\ s\right]}{\sqrt{\pi}}$$

■ 4.2

Define a list of functions with alpha = 0, 0.1, 0.2, ..., 1 using Table [ ],

```
funcs = Table[indefInt, {alpha, 0, 1, .1}]
```

$$\left\{\frac{\mathrm{Erf}[0.\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.316228\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.447214\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.547723\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.632456\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.707107\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.774597\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.83666\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.894427\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[0.948683\ s]}{\sqrt{\pi}}, \frac{\mathrm{Erf}[1.\ s]}{\sqrt{\pi}}\right\}$$

Plot the list of 11 functions within -5 < s < 5,

```
Plot[funcs, {s, -5, 5}]
```