3.016 Problem Set #1, 2010

I1-1

Define the function fOfX, and let Mathematica express it

\[
\text{fOfX} = \frac{1}{(1/2 + \sin[x])^x}
\]

Take derivative of the function fOfX, and let Mathematica simplify the result

\[
\text{Simplify[D[fOfX, x]]}
\]

I1-2

2.1

Assign \( x = (1+y) \)

\[
x = (1+y);
\]

Expand \( x^2 \) and \( x^4 \)

\[
\text{Expand[(1+y)^2]}
\]
\[
\text{Expand[(1+y)^4]}
\]

2.2

Define the variable \( z \)

\[
z = \text{Exp[Integrate[(s/(1+s^2)), {s, 0, x^2}, Assumptions \rightarrow y \in \text{Reals}]]}
\]

\[
\sqrt{2 + y (2 + y) (2 + y (2 + y))}
\]
Try simplifying the result

```
Simplify[z, Assumptions \[\in\] Reals]
\[\sqrt{2 + y (2 + y) (2 + y (2 + y))}\]
```

Factor the result

```
Factor[z]
\[\sqrt{2 + 4 y + 6 y^2 + 4 y^3 + y^4}\]
```

Verify that \[\sqrt{1 + (1 + y)^4}\] is equivalent to \[2 + 4 y + 6 y^2 + 4 y^3 + y^4\]

```
Expand[1 + (1 + y)^4]
2 + 4 y + 6 y^2 + 4 y^3 + y^4
```

```
Sqrt[1 + (1 + y)^4]
\[\sqrt{1 + (1 + y)^4}\]
```

### 2.3

Take the derivative as required, result is assigned to dThing

```
dThing = D[(1 + x)^z, y]
(2 + y) \[\sqrt{2 + y (2 + y) (2 + y (2 + y))}\] \[\frac{\sqrt{2 + y (2 + y) (2 + y (2 + y))}}{2 + y}\] + \\
(2 + y) (2 + y) (2 + y) (2 + y) + y (2 + y (2 + y)) + (2 + y) (2 + y (2 + y)) \[\text{Log}[2 + y]\] \\
\frac{2 \sqrt{2 + y (2 + y) (2 + y (2 + y))}}{2 + y (2 + y) (2 + y)}
```
2.4

Plot the above derivative \( d \text{Thing} \) within \(-3 < y < 0\)

\[
\text{Plot}[d \text{Thing}, \{y, -3, 0\}]
\]

Interesting very flat minimum between \(-1.5\) and \(-0.5\), explore nearby functional behavior...

\[
\text{Plot}[d \text{Thing}, \{y, -1.25, -0.5\}]
\]

\[
\text{FindMinimum}[d \text{Thing}, \{y, -1\}]
\]

FindMinimum::fmgz:
Encountered a gradient that is effectively zero. The result returned may not be a minimum; it may be a maximum or a saddle point. 

\[
\{1., \{y \rightarrow -1.\}\}
\]

Must be a very flat minimum. See if we can verify the minimum at \( y = -1 \).

\[
\text{D}[d \text{Thing}, y] \/. y \rightarrow -1
\]

0
\[ d\text{Thing} \cdot y \rightarrow -1 \]

1

11-3

3.1

Define \texttt{aFunc} that returns a list with variables \( x \) and \( n \),

\[
\texttt{aFunc}[x_, n_] := \text{Table}[x^i, \{i, 0, n\}] 
\]

Obtain the list using \texttt{aFunc} with \( x = 2 \) and \( n = 12 \),

\[
\texttt{aFunc}[2, 12] \\
\{1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096\} 
\]

3.2

Divide every member of the list by 2,

\[
\texttt{aFunc}[2, 12] / 2 \\
\frac{1}{2} \\
\{1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048\} 
\]

The above is a list of \( 2^i \) for \( i = -1, 0, 1, 2, \ldots, n-2 \). Now doing the substraction,

\[
\texttt{aFunc}[2, 12] - \texttt{aFunc}[2, 12] / 2 \\
\{1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048\} 
\]

This makes sense because each member is \( 2^i - 2^{i-1} = (1-1/2) 2^i = 2^i (1-1/2) \).

3.3

Simply evaluate \texttt{aFunc} with \( x = \text{Carter} \) and \( n = 100 \) will give the desired list,

\[
\texttt{aFunc}[\text{Carter}, 100] \\
\{1, \text{Carter}, \text{Carter}^2, \text{Carter}^3, \text{Carter}^4, \text{Carter}^5, \text{Carter}^6, \text{Carter}^7, \text{Carter}^8, \text{Carter}^9, \text{Carter}^{10}, \text{Carter}^{11}, \text{Carter}^{12}, \text{Carter}^{13}, \text{Carter}^{14}, \text{Carter}^{15}, \text{Carter}^{16}, \text{Carter}^{17}, \text{Carter}^{18}, \text{Carter}^{19}, \text{Carter}^{20}, \text{Carter}^{21}, \text{Carter}^{22}, \text{Carter}^{23}, \text{Carter}^{24}, \text{Carter}^{25}, \text{Carter}^{26}, \text{Carter}^{27}, \text{Carter}^{28}, \text{Carter}^{29}, \text{Carter}^{30}, \text{Carter}^{31}, \text{Carter}^{32}, \text{Carter}^{33}, \text{Carter}^{34}, \text{Carter}^{35}, \text{Carter}^{36}, \text{Carter}^{37}, \text{Carter}^{38}, \text{Carter}^{39}, \text{Carter}^{40}, \text{Carter}^{41}, \text{Carter}^{42}, \text{Carter}^{43}, \text{Carter}^{44}, \text{Carter}^{45}, \text{Carter}^{46}, \text{Carter}^{47}, \text{Carter}^{48}, \text{Carter}^{49}, \text{Carter}^{50}, \text{Carter}^{51}, \text{Carter}^{52}, \text{Carter}^{53}, \text{Carter}^{54}, \text{Carter}^{55}, \text{Carter}^{56}, \text{Carter}^{57}, \text{Carter}^{58}, \text{Carter}^{59}, \text{Carter}^{60}, \text{Carter}^{61}, \text{Carter}^{62}, \text{Carter}^{63}, \text{Carter}^{64}, \text{Carter}^{65}, \text{Carter}^{66}, \text{Carter}^{67}, \text{Carter}^{68}, \text{Carter}^{69}, \text{Carter}^{70}, \text{Carter}^{71}, \text{Carter}^{72}, \text{Carter}^{73}, \text{Carter}^{74}, \text{Carter}^{75}, \text{Carter}^{76}, \text{Carter}^{77}, \text{Carter}^{78}, \text{Carter}^{79}, \text{Carter}^{80}, \text{Carter}^{81}, \text{Carter}^{82}, \text{Carter}^{83}, \text{Carter}^{84}, \text{Carter}^{85}, \text{Carter}^{86}, \text{Carter}^{87}, \text{Carter}^{88}, \text{Carter}^{89}, \text{Carter}^{90}, \text{Carter}^{91}, \text{Carter}^{92}, \text{Carter}^{93}, \text{Carter}^{94}, \text{Carter}^{95}, \text{Carter}^{96}, \text{Carter}^{97}, \text{Carter}^{98}, \text{Carter}^{99}, \text{Carter}^{100}\} 
\]
4.1

Evaluate the indefinite integral as required,

\[ \text{indefInt} = \left( 2 \sqrt{\text{alpha}} / \pi \right) \text{Integrate}[\text{Exp}\left[-\text{alpha} s^2\right], s] \]

\[ \frac{\text{Erf} \left[ \sqrt{\alpha} s \right]}{\sqrt{\pi}} \]

4.2

Define a list of functions with \( \alpha = 0, 0.1, 0.2, \ldots, 1 \) using Table []

\[ \text{funcs} = \text{Table}[\text{indefInt}, \{\text{alpha}, 0, 1, .1\}] \]

\[ \frac{\text{Erf}[0. s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.316228 s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.447214 s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.547723 s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.632456 s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.707107 s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.774597 s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.83666 s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.894427 s]}{\sqrt{\pi}}, \frac{\text{Erf}[0.948683 s]}{\sqrt{\pi}}, \frac{\text{Erf}[1. s]}{\sqrt{\pi}} \] \]

Plot the list of 11 functions within \(-5 < s < 5\),

\[ \text{Plot}[	ext{funcs}, \{s, -5, 5\}] \]