

# Divergence and Curl and Their Geometric Interpretations

1

## Scalar Potentials: Their Gradient Fields and Visualization

2

## Visualizing Gradient Fields and Laplacian of a Scalar Potential

*Gradient field of three-hole potential*

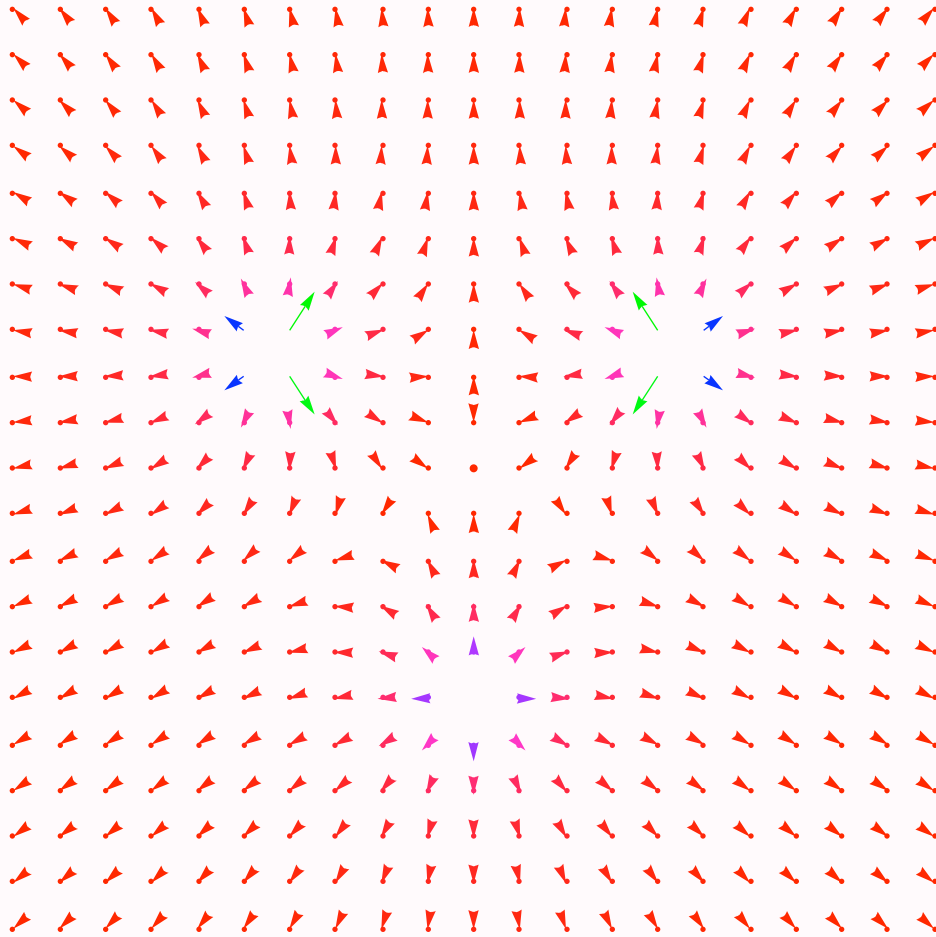
```
gradthreehole = gradfield[ThreeHolePotential]
```

1

$$\left\{ \begin{aligned} & \frac{-\frac{\sqrt{3}}{2} + x}{\left(1 - \sqrt{3} x + x^2 - y + y^2\right)^{3/2}} + \frac{\frac{\sqrt{3}}{2} + x}{\left(1 + \sqrt{3} x + x^2 - y + y^2\right)^{3/2}} + \frac{x}{\left(x^2 + (1 + y)^2\right)^{3/2}}, \\ & \frac{-\frac{1}{2} + y}{\left(1 - \sqrt{3} x + x^2 - y + y^2\right)^{3/2}} + \frac{-\frac{1}{2} + y}{\left(1 + \sqrt{3} x + x^2 - y + y^2\right)^{3/2}} + \frac{1 + y}{\left(x^2 + (1 + y)^2\right)^{3/2}} \end{aligned} \right\}$$

```
Needs["VectorFieldPlots`"];
VectorFieldPlots`VectorFieldPlot[gradthreehole, {x, -2, 2}, {y, -2, 2},
ScaleFactor -> 0.2, ColorFunction -> (Hue[1 - #1 0.66] &), PlotPoints -> 21]
```

2



*Function that takes a two-dimensional vector function of (x,y) as an argument and returns its divergence*

```
divergence[{xcomp_ , ycomp_}] := Simplify[D[xcomp, x] + D[ycomp, y]]
```

3

```
divgradthreehole = divergence[gradfield[ThreeHolePotential]] // Simplify
```

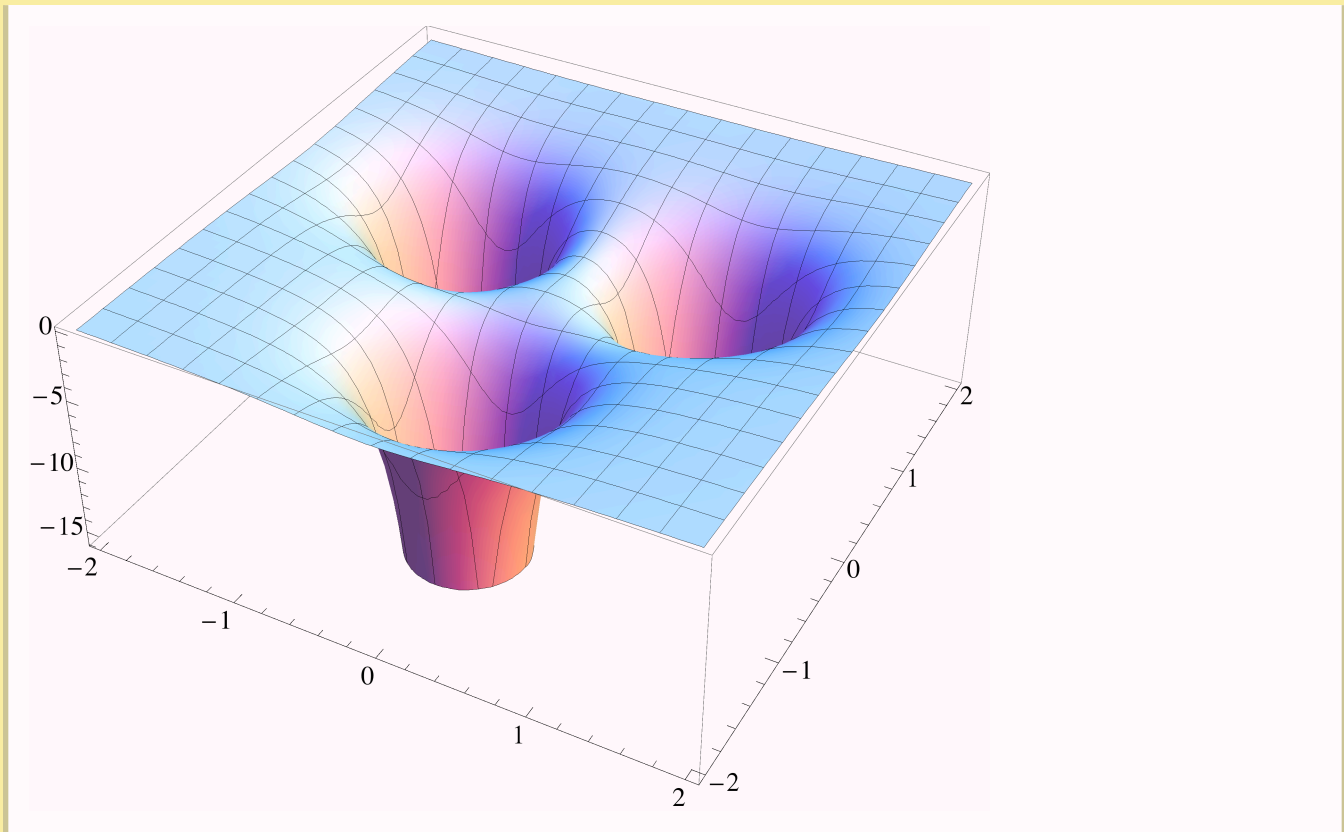
4

$$\begin{aligned}
 & -\frac{3(\sqrt{3}-2x)^2}{4(1-\sqrt{3}x+x^2-y+y^2)^{5/2}} - \frac{3(1-2y)^2}{4(1-\sqrt{3}x+x^2-y+y^2)^{5/2}} + \\
 & \frac{2}{(1-\sqrt{3}x+x^2-y+y^2)^{3/2}} - \frac{3(\sqrt{3}+2x)^2}{4(1+\sqrt{3}x+x^2-y+y^2)^{5/2}} - \frac{3(1-2y)^2}{4(1+\sqrt{3}x+x^2-y+y^2)^{5/2}} + \\
 & \frac{2}{(1+\sqrt{3}x+x^2-y+y^2)^{3/2}} - \frac{3x^2}{(x^2+(1+y)^2)^{5/2}} - \frac{3(1+y)^2}{(x^2+(1+y)^2)^{5/2}} + \frac{2}{(x^2+(1+y)^2)^{3/2}}
 \end{aligned}$$

Plotting the divergence of the gradient ( $\nabla \cdot (\nabla f)$  is the "Laplacian"  $\nabla^2 f$ , sometimes indicated with symbol  $\Delta f$ )

```
Plot3D[divgradthreehole, {x, -2, 2}, {y, -2, 2}, PlotPoints -> 60]
```

5



3

**Coordinate Transformations in the Vector Analysis Package**

4

**Coordinate Transforms Example: Calculating Distances between Two Cities**

5

**Using Vector Derivative Functions in the Vector Analysis Package**

6

**A Visualization Example of the Curl**