Lecture 1: Introduction and Course Description

———————————————————— Sept. 9 2009 ————————————————————

These notes and all course materials will be available at http://pruffle.mit.edu/3.016-2009. Students should bookmark this site and use it to download lecture notes, homework, and reading assignments for laboratories and lectures. The site will change throughout the semester.

The web materials for 3.016 are revised each year. Previous years' notes, homeworks, and laboratories are available at http://pruffle.mit.edu/3.016-2008 and may be useful to you.

There are a few things you should do immediately:

- If you don't have your own laptop, then let me know and we can use MIT's laptop loaner program: http://web.mit.edu/lcp/.

- Get a copy of mathematica and a student user license: http://web.mit.edu/is/products/vsls/. There is a minimal fee for your student license, but it is also freely available through Athena.

- Read MIT's policy on academic integrity (html) or (pdf). Print out the first page and sign it—this will imply that you have read it. You will not pass this course unless you do this.

Preface

Materials Science and Engineering is a discipline that combines knowledge of chemistry, mechanics, and physics and then applies them to the study of materials and their properties. It is a challenging and diverse enterprise—obtaining expertise in a large set of diverse subjects—but is one that will be very rewarding and fulfilling.

Mathematics is the *language* that binds together disparate topics in physics, engineering, and chemistry. Thus, this subject is for undergraduate materials scientists and engineers who wish to learn about the mathematics that is essential to their chosen field.

While it is possible to become an excellent materials scientist and engineer without some working knowledge of a large subset of mathematical topics, it is much easier to master this discipline with mathematics to guide you. Through mathematics, you will discover that some topics have similarities that are not obvious and, in fact, are not taught to you as *being* similar. Such similarities and analogies will make learning much, much easier—and I believe much more enjoyable.

MIT's Department of Materials Science and Engineering (MS&E) subjects have been designed with the philosophy that students will benefit more from a solid background in, and a working knowledge of, the wide range math problem solving techniques that pertain specifically to MS&E, rather than a limited subset of topics (albeit at more depth and rigor) as taught in a one-semester subject in a Mathematics department. It is reasonable to ask, "Is this subject a substitute for a a 'Linear Algebra' or 'Partial Differential Equations' from the Math department?" The answer is, "no, not entirely." This subject is not a replacement for a single math subject that provides the rigorous foundations of Mathematics, and I encourage you to take such math classes in the future if you have time. This subject is designed to be very broad in scope and therefore its depth in any one topic is limited. However, you will learn to use *mathematics as a tool* to solve engineering problems in this course. And, you will learn math by applying it to familiar problems. I believe it will be easier and more interesting to take a Math department course *after* you finish 3.016.

I do believe very strongly that you will enjoy studying math more after taking this introduction and that the mathematical background you will receive this semester will make your Materials Science education richer and more rewarding.

I have designed this subject to help you learn as much essential math as possible in a short time. To this end, this subject has several unusual aspects that you will need to know.

3.016 Mathematical Software

Symbolic mathematical computer software is a tool used by almost every applied scientist. Such software helps produce results quickly, visualizes and documents the results, and minimizes the silly errors that creep into complicated mathematical manipulations. Although there are many other good choices, I have decided to use MATHEMATICA® as a vehicle for learning and doing mathematics. It has a fairly steep learning curve. Once learned however, it rewards the time investment with a powerful tool that, though not a replacement for mathematical understanding, will serve as an aid to help you think about, visualize, compute, and solve mathematics problems faster and more accurately.

MATHEMATICA® is available for all MIT students, both on Athena (free) and via licenses for personal laptop and desktop machines (There is a modest fee, $30, for student licenses, but it is a useful investment for other subjects. The license is for one year and will expire on June 30, 2010). To purchase and download MATHEMATICA® , you should go to http://ist.mit.edu/services/software/mathematica/obtain and scroll to the bottom of that page. If you have Mac or Windows operating systems, you will be able to download the software. If you are running linux, then you will need to pick up a DVD in building N42.

You will need MATHEMATICA® for your first homework set and laboratory, and you should try to get it loaded and working very soon, preferably by the first weekend after classes start. If you have a laptop, I suggest that you install MATHEMATICA® on it as soon as possible. If you don't have a laptop, I can write you a letter so that you can get one through MIT's laptop-loaner program: http://web.mit.edu/lcp/.

Note, the purpose of this course is not to teach MATHEMATICA® . I will teach you how to use it as a tool to learn and solve problems. Thus, you will have a fairly good working knowledge of the software and will have the elements—if you invest the time and work—to become a MATHEMATICA® power-user in the future. In this course, you will learn enough basics so that you be able to solve engineering problems faster, more accurately, and more beautifully, than your MATHEMATICA® -naive cohorts. I will teach some power-user methods in the recitation sections.

There are useful tutorials on the web. I recommend that you spend some time watching these useful "Hands On Introduction to MATHEMATICA® " at http://url.wolfram.com/42xH_m/, http://url.wolfram.com/3yYhKE/, and http://url.wolfram.com/3_dIko/.

Laboratories in this course will be performed using MATHEMATICA® . Laboratory assignments must be completed during the laboratory period and an electronic copy of a MATHEMATICA® notebook for each laboratory must be emailed to the instructor and the TA.

3.016 Examination Philosophy

Tests and exams are powerful motivators to get students to take a subject seriously, but I believe that working through homework problems better promotes learning, particularly for self-motivated students.

Therefore, there will be no written exams, tests, or quizzes in 3.016. Your grade will be based on your homeworks, laboratories, participation in class and recitations, and oral exams given during the recitation. The homeworks and the laboratories will be graded carefully (described below). A homework calendar is given below.

3.016 Homework

The purpose of the homework is to help you solidify your understanding of *mathematics applied to engineering and science problems* by working through examples. Some examples will be exercises in mathematics; others will be exercises in application of mathematics for solving engineering and science problems. I encourage you to use MATHEMATICA® to solve your homework problems, and you may turn in solutions as printed MATHEMATICA® notebooks. Nevertheless to appreciate what symbolic mathematics programs can do for you, there will be some exercises that I will ask you to do with pencil

and paper. Nevertheless, there is no harm in checking your "by-hand" results with MATHEMATICA®
.

When you do homework, you are not under the potentially menacing eyes of an exam proctor. This means that you can receive help in the form through various resources, by asking experts, or interacting with fellow students.

Resources Go to the library, or refer other media to find solutions to problems. It is good practice and you will learn quite a bit by doing so. I recommend that you first attempt to find a solution before going to the library—not only will it help you appreciate the solution, it will also make your search a bit easier! *However, if your solutions derive from any source that you have found, then you must cite that source.* **Plagiarists will receive NO CREDIT for their work. Also, because I can't tell who is plagarizing and who is the source the following policy will be enforced: if any two or more handed-in homework-assignments appear to be duplicated without proper citation, then ALL of the homework-assignments will receive NO CREDIT.** This applies to the "original document" as well.

Experts By all means, consult with experts on your homework. It is a good idea to do this as long as you understand what *you* turn in.

Classmates This is the best choice of all. I think it is both inevitable and beneficial to give and receive help. Cooperating on homework will help you learn to communicate your ideas and begin to appreciate the difficulties and rewards of teamwork.

However, the work you turn in must be your own, or properly cited.

As explained below, the homework assignments in 3.016 will be, in part, cooperatively oriented.

You will find that you are more busy some weeks than others and relying on a classmate during a busy week can be a life-saver. However, if you start slacking off and don't hold up your end of the bargain when you are able, you will engender resentment, as well as endanger professional and friendly relationships. I leave it to your own conscience to play fairly and contribute when you can and, while understanding that everyone experiences different kinds of pressures, to be forthright and honest with others who do not contribute consistently.

However, *you must contribute something to every problem on every group assignment. If you don't contribute, you must state it. If I find out later that you didn't contribute to a problem, but took credit for it, then you will receive NO CREDIT for the entire assignment.*

It is fairly easy for the instructor to ascertain who is slacking and who is not. I can't say that my good opinion has any particular value; but keep in mind, that slackers may have hard time regaining my good opinion.

Homework cooperation has a potential downside because you all receive individual grades. We will attempt to mitigate this downside by dividing the homework into two parts:

Group For each homework set, one or more multi-part problems will be designated as *Group Exercises*. For these, the entire group will turn in *one homework*. Every member of the group *who puts their name on the turned-in assignment* will receive exactly the same credit for the homework grade (unless it has been determined later that the student did not contribute).

Homework groups will be *assigned* with each homework set. The groups will change from week to week and the members will be assigned randomly. ***POLL*** Each group will be assigned a homework leader who will be responsible for arranging meetings and turning in the homework.

For each group problem, I suggest that the students agree to divide up various tasks. For example, one student would develop the model, one would work out the mathematical approach, another would develop the MATHEMATICA® code, another should comment the code, another student should be responsible for writing descriptions and interpretations of the results, and another student should be responsible for improving the graphics.

By putting each individual's name on a homework assignment, the group verifies that each indicated person has contributed to the assignment. By putting your own name on the group's turned-in assignment, indicates that you have reviewed *all* of the assignment; if questioned, each person should be able to describe how each problem was done. MIT's policy on academic integrity is also the policy for 3.016.

I *might* introduce a mechanism where students can indicate who is contributing to group homeworks and who is not. If introduced, this mechanism will also affecting grading and the composition of groups.

Individual Each problem set will contain a few problems for each student to complete individually. These problems will come out of the textbook or will be constructed by the instructor. The individual problems will be easier than the group exercises. They are designed to maintain a sufficient amount of currency and emphasize that reading the textbook is an essential part of this course. The problems will demonstrate the benefits of having symbolic mathematics software.

If you turn in work that you did not do, and do not attribute the solution to its rightful author(s), then you are plagiarizing. As a first assignment in this course, every one of you should read MIT's policy on academic integrity (html) or (pdf) immediately. There is also an on-line learning module http://web.mit.edu/uaap/learning/modules/acadintegrity/ to help you understand the philosophy of integrity and your responsibility as an MIT student.

3.016 Laboratory

There will be a 3.016 laboratory each thursday that 3.016 meets (see calendar below). The labs will be practical and will focus on using MATHEMATICA® effectively.

There will be assigned readings from the MATHEMATICA® help browser that comes with the software. *You should always do this reading before the laboratory, or you may not be able to* FINISH YOUR ASSIGNMENT AND TURN IT IN BEFORE THE END OF THE LABORATORY IN ORDER TO GET CREDIT. You will be able to get help from the instructor, or your classmates during the laboratory. However, if it becomes apparent that you did not do the reading, the instructor will not give you help.

If you stay current in the course material and keep up with the homework assignments, you should have no difficulty doing the laboratory assignments *if you do the pre-assigned reading.*

You should have your own laptop running MATHEMATICA® and bring it to the lab with you. Laptop loaners from MIT computing services are available.

Grades

As stated above, all of the final grade will depend on the homework, the laboratory assignments, class participation, group contributions, and oral exams. There is no fixed average grade for this course; the average will depend on the entire class performance. However, if your homework grades and your laboratory reports are consistently within the top quartile, then it is extremely likely that you will receive an **A**.

Late homeworks will be penalized by 10% (compounded daily) of the total score for the homework. Homeworks will be evaluated on the basis of:

Accuracy The solution must be a reasonable and correct answer to the homework question.

Exposition The solution must clearly show the reasoning that was utilized to find it and the method of solution should be clearly apparent. Exegetic solutions will be ranked higher.

Beauty Good solutions will often require graphics that, with care, can beautifully explain the solution. The layout of the page, the quality of the supporting prose, the clarity of the graphics, and all

that "je ne sais quoi" is fairly subjective but very important. The grader will include a judgment of your craft and art.

Observation Supplemental observations provide aids in understanding and demonstrate mastery of a topic. An example of a supplemental observation might be something like, "Note that in the limit of long times, that the total concentration goes to zero. This is sensible because the boundary condition on mass flux is directed outward everywhere on the finite domain."

Laboratories will be graded on their completeness, demonstrated mastery of MATHEMATICA® for that assignment, and exposition.

Grade Calculations

Your individual assignments will be graded by ranking them from *Best* to *Least Best Homework*. A decision will be made regarding how many points (out of a possible 50) the *Least Best Homework* deserves, and the homework scores will then be interpolated between a score of 50 for the *Best Homework* and that of the *Least Best*.

Furthermore, for group assignments, the Score$_{\text{as graded}}$ will be computed as a sum *and a product* of each of the multi-part questions as follows:

$$\text{Score}^{\text{group}}_{\text{as graded}} = \frac{1}{2}\left(\frac{\sum_i^N \text{Score}_{\text{Subproblem}_i}}{\sum_i^N \text{Score}^{\max}_{\text{Subproblem}_i}}\right) + \frac{1}{2}\left(\frac{\prod_i^N \text{Score}_{\text{Subproblem}_i}}{\prod_i^N \text{Score}^{\max}_{\text{Subproblem}_i}}\right)$$

Thus, a very low score (e.g., zero) on any one part of the multipart problem will have a very large effect on the group problem final grade.

Homework Calendar

Homework Schedule		
Homework	**Out**	**In**
Set 1	9 Sept.	18 Sept.
Set 2	18 Sept.	5 Oct.
Set 3	3 Oct.	19 Oct.
Set 4	15 Oct.	26 Oct.
Set 5	26 Oct.	16 Nov.
Set 6	20 Nov.	7 Dec.

Late Policy

Laboratory assignments must be turned in during the laboratory period. You must show documentation of unforeseeable emergencies that prevent you from attending a laboratory period. Any missed laboratories must be made up by special arrangement. If for some reason, you cannot complete a laboratory during the laboratory period, you should send a paragraph explaining why you could not finish.

It is your responsibility to do the assigned reading before the laboratory.

Textbook

We will use a fairly general textbook on applied mathematics (E. Kreyszig, *Advanced Engineering Mathematics*, ninth ed., J.W. Wiley, ≈ 1200 pages). You'll notice that reading assignments do not follow the table of contents—while I like the book, there are pedagogical reasons for studying mathematics in the sequence I chose to follow in this subject. Extra material pertaining to materials science specifically will be created and placed on the web.

I have identified 66 sections of the book (330 pages in total) for required reading. The readings for each lecture will appear in the Lecture Notes and posted on the course web-site at: http://pruffle.mit.edu/3.016-2009. I hope you will keep up with the reading—I think it would be wise to give the material a cursory reading prior to the lecture and then read it more carefully before starting the homework.

This course is designated as a 12 (*3-1-8*) unit subject[1] Time spent awake during lectures and recitations is less than half of your job—reading and doing homework comprise the greater part.

Lecture Notes

Lecture notes (like these) will be available for you to print out for each lecture. The lecture notes will be available at: http://pruffle.mit.edu/3.016-2009. These will supplement (not replace) the textbook. The lecture notes also serve as a guide to help the student understand what parts of the text are considered more relevant or important.

The specific purpose of the notes is to provide neatly typeset equations and graphics that will be used in the lecture along with a few observations. This will eliminate the time required to write and draw, perhaps a bit sloppily, for you in your notes and for me on the blackboard.

The lecture notes will have reading assignments printed at the beginning of each lecture; they will look like this:

Kreyszig **6.1, 6.2, 6.3, 6.4** (pages: 304–309, 312–318, 321–323, 331–336). Part of the units for this course involve reading. You are receiving an expensive education—you should strive to make your education valuable by doing all the required reading. Your intellect will profit even more by doing outside reading.

Those concepts that are fundamental to this course will be presented in lectures by the lecturer (or in the form of welcome questions and points of clarification by the students) and some explanatory notes will be written upon the blackboard.

You are responsible for the reading materials from the textbook and the lecture notes. Occasionly, the instructor will introduce extra material during the lecture that will displace topics in the notes.

The notes will have places for you to fill in auxiliary discussion and explanation. Those places will look like this:

You can use these notes in several ways. You could print them out before lecture, and write your own lecture notes directly on them during the lecture. You could take lecture notes on your own paper and then neatly copy them onto a printout later. You could print them before lecture and write on them rapidly and then copy—neatly and thoughtfully—your notes onto a freshly printed set. I recommend the latter for effective learning and the creation of a set of notes that might provide future reference material—but do whatever works for you.

The lecture notes will also refer to MATHEMATICA® notebooks available on the 3.016 website for downloading. These notebooks will be used as MATHEMATICA® sessions during the lectures to illustrate specific points and provide examples for you to help solve homework problems.

References to MATHEMATICA® notebooks look like the ones given at the end of this lecture's notes in section 1.

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[1] Units at MIT are assigned under the following schema: *lec-lab-out* where *lec* is the number of lecture/recitation hours, *lab* is the number of laboratory hours, and *out* is the number of outside (reading, preparation, homework) hours per week. One MIT unit represents about 14 hours of semester work on the average.

These examples will serve as place-holders in the lecture note when we switch from chalkboard and/or projected display of the notes to a live MATHEMATICA® session.

Lecture and Laboratory Calendar

This calendar will be updated throughout the semester. Students should consult this calendar weekly to obtain the required reading assignments for the laboratory.

Week of 7—11 September

Lectures		
	Topics	**Reading**
M 09/07	Labor Day, No Lectures	
W 09/09 Lect. 1	Course organization and introduction to Mathematica, Common Errors for Beginners	Course Notes and Mathematica Notebook *I*
F 09/11 Lect. 2	Introduction to Mathematica, assignment and evaluation, rules and replacement, basic calculus and plotting, lists and matrices, getting help	Course Notes and Mathematica Notebook *II*

Laboratory		
Date	**Topics**	**Reading**
Th 09/10 Lab 0	Discussion of Labotory and Expectations	*Start Reading for Laboratory 1 as Soon as Possible.*

Homework		
Homework Set	Out	Due
1	**9 Sept.**	18 Sept.

Week of 14—18 September

Lectures		
	Topics	**Reading**
M 09/14 Lect. 3	Mathematica programming: functions and patterns, localized variables, logical switches, recursion; Graphics: plotting lists of data, examples	Course Notes and Mathematica Notebook *III*
W 09/16 Lect. 4	Mathematica: symbolic and numerical operations, operations on expressions, solving equations, numerical solutions, file input and output, using packages	Course Notes and Mathematica Notebook *IV*
F 09/18 Lect. 5	Mathematica: overview of graphics, animation, interaction, graphics primitives, complete worked examples	Course Notes and Mathematica Notebook *V*

Laboratory		
Date	**Topics**	**Reading**
Th 09/17 Lab 1	Symbolic calculations, calculus and plotting	*Mathematica Help Browser* **"First Five Minutes with Mathematica,"** **"tutorial/YourFirstMathematicaCalculations,"** **"tutorial/AlgebraicCalculationsOverview."** **"tutorial/BasicEditingTechniquesOverview,"** **"tutorial/ListsOverview,"** **"tutorial/DefiningFunctions,"** **"tutorial/BasicPlotting,"** **"tutorial/Options"** http://www.wolfram.com/broadcast/#Tutorials-GS **Functions: Integrate, Table, Simplify, NIntegrate, Plot, Plot3D, ContourPlot**

Homework		
Homework Set	Out	Due
1	9 Sept.	**18 Sept.**
2	**18 Sept.**	27 Sept.

Week of 21—25 September

3.014 Laboratory Week: 3.016 does not meet.

Week 28 Sept.—2 October

Lectures		
	Topics	**Reading**
M 09/28 Lect. 6	Linear algebra: matrix operations, interpretations of matrix operations, multiplication, transposes, index notation	*Kreyszig* **7.1, 7.2, 7.3, 7.4** (pages: 272–276, 278–286, 287–294, 296–301)
W 09/30 Lect. 7	Linear algebra: solutions to linear systems of equations, determinants, matrix inverses, linear transformations and vector spaces	*Kreyszig* **7.5, 7.6, 7.7, 7.8, 7.9** (pages: 302–305, 306–307, 308–314, 315–323, 323–329)
F 10/02 Lect. 8	Complex numbers: complex plane, addition and multiplication, complex conjugates, polar form of complex numbers, powers and roots, exponentiation, hyperbolic and trigonometric forms	*Kreyszig* **13.1, 13.2, 13.5, 13.6** (pages: 602–606, 607–611, 623–626, 626–629)

Laboratory		
Date	**Topics**	**Reading**
Th 10/01 Lab 2	Solving linear systems of equations	*Mathematica Help Browser* **"tutorial/ConstructingMatrices," "tutorial/GettingAndSettingPiecesOfMatrices," "tutorial/OperationsOnScalarsVectorsAndMatrices," "tutorial/MultiplyingVectorsAndMatrices," "tutorial/VectorOperations," "tutorial/MatrixInversion," "tutorial/BasicMatrixOperations," "tutorial/SolvingLinearSystems," "tutorial/EigenvaluesAndEigenvectors";** **Functions: Solve, Inverse, Transpose, Eigensystem**

Homework		
Homework Set	Out	Due
3	**2 Oct.**	19 Oct.

Week of 5—9 October

Lectures		
	Topics	**Reading**
M 10/05 Lect. 9	Matrix eigenvalues: eigenvalue/eigenvector definitions, invariants, principal directions and values, symmetric, skew-symmetric, and orthogonal systems, orthogonal transformations	*Kreyszig* **8.1, 8.2, 8.3** (pages: 334–338, 340–343, 345–348)
W 10/07 Lect. 10	Hermitian forms, similar matrices, eigenvalue basis, diagonal forms	*Kreyszig* **8.4, 8.5** (pages: 349–354, 356–361)
F 10/09 Lect. 11	Vector calculus: vector algebra, inner products, cross products, determinants as triple products, derivatives of vectors	*Kreyszig* **9.1, 9.2, 9.3, 9.4** (pages: 364–369, 371–374, 377–383, 384–388)

Laboratory		
Date	**Topics**	**Reading**
Th 10/8 Lab 3	File input/output, plotting data	*Mathematica Help Browser* **"tutorial/ImportingAndExportingFiles," "tutorial/ImportingAndExportingData," "tutorial/lottingListsOfData," "tutorial/ManipulatingNumericalData," "tutorial/CurveFitting," "guide/Statistics";** **Functions: Dimensions, Append, AppendTo, Mean, StandardDeviation, ListPlot, Table, MultipleListPlot, Fit, FindFit**

Homework		
Homework Set	Out	Due
2	18 Sept.	**5 Oct.**

Week of 12—16 October

Lectures		
	Topics	**Reading**
T 10/13 Lect. 12	Multi-variable calculus: curves and arc length, differentials of scalar functions of vector arguments, chain rules for several variables, change of variable and thermodynamic notation, gradients and directional derivatives	*Kreyszig* **9.5, 9.6, 9.7** (pages: 389–398, 400–403, 403–409)
W 10/14 Lect. 13	Vector differential operations: divergence and its interpretation, curl and its interpretation	*Kreyszig* **9.8, 9.9** (pages: 410–413, 414–416)
F 10/16	3.014 Meets, No Lecture	

Laboratory		
Date	**Topics**	**Reading**
F 10/15 Lab 4	Graphical representations in three and higher dimensions	*Mathematica Help Browser* **"tutorial/BasicPlotting," "tutorial/RedrawingAndCombiningPlots," "tutorial/ThreeDimensionalSurfacePlots," "tutorial/ParametricPlots," "tutorial/SomeSpecialPlots,"**

Homework		
Homework Set	Out	Due
4	**15 Oct.**	26 Oct.

Week of 19—23 October

3.014 Laboratory Week: 3.016 does not meet.

Homework		
Homework Set	Out	Due
3	2 Oct.	**19 Oct.**

Week of 26—30 October

Lectures		
	Topics	**Reading**
M 10/26 Lect. 14	Path integration: integral over a curve, change of variables, multidimensional integrals	*Kreyszig* **10.1, 10.2, 10.3** (pages: 420–425, 426–432, 433–439)
W 10/28 Lect. 15	Multidimensional forms of the Fundamental theorem of calculus: Green's theorem in the plane, surface representations and integrals	*Kreyszig* **10.4, 10.5, 10.6, 10.7** (pages: 439–444, 445–448, 449–458, 459–462)
F 10/30 Lect 16	Multi-variable calculus: triple integrals and divergence theorem, applications and interpretation of the divergence theorem, Stokes' theorem.	*Kreyszig* **10.8, 10.9** (pages: 463–467, 468–473)

Laboratory		
Date	**Topics**	**Reading**
Th 10/29 Lab 5	Multivariable Calculus	*Mathematica Help Browser* **"tutorial/Differentiation," "VectorAnalysis/tutorial/VectorAnalysis," "VectorAnalysis/guide/VectorAnalysisPackage,"**

Homework		
Homework Set	Out	Due
4	15 Oct.	**26 Oct.**
5	**26 Oct.**	16 Nov.

Week of 2—6 November

Lectures		
	Topics	**Reading**
M 11/2 Lect. 17	Periodic functions: Fourier series, Interpretation of Fourier coefficients, convergence, odd and even expansions	*Kreyszig* **11.1, 11.2, 11.3** (pages: 478–485, 487–489, 490–495)
W 11/4 Lect. 18	Fourier theory: complex form of Fourier series, Fourier integrals, Fourier cosine and sine transforms, the Fourier transforms	*Kreyszig* **11.4, 11.7, 11.8, 11.9** (pages: 496–498, 506–512 513–517, 518–523)
F 11/6 Lect 19	Ordinary differential equations: physical interpretations, geometrical interpretations, separable equations	*Kreyszig* **1.1, 1.2, 1.3** (pages: 2–8, 9–11, 12–17)

Laboratory		
Date	**Topics**	**Reading**
Th 11/05 Lab 6	Optimization	*Mathematica Help Browser* **"tutorial/NumericalOptimization";** **Functions: Minimize, Maximize, FindMinimum**

Week of 9—13 November

3.014 Laboratory Week: 3.016 does not meet.

Week of 16—20 November

Lectures		
	Topics	**Reading**
M 11/16	3.014 Meets, No Lecture	
W 11/18 Lect. 20	ODEs: derivations for simple models, exact equations and integrating factors, the Bernoulli equation	*Kreyszig* **1.4**, **1.5** (pages: 19–25, 26–32)
F 11/20 Lect. 21	Higher order differential equations: homogeneous second order, initial value problems, second order with constant coefficients, solution behavior	*Kreyszig* **2.1**, **2.2** (pages: 45–52, 53–58)

Laboratory		
Date	**Topics**	**Reading**
Th 11/19 Lab 7	Solving Differential Equations	*Mathematica Help Browser* **"tutorial/DifferentialEquations,"** **"tutorial/IntroductionToNumericalDifferentialEquations,"** **"tutorial/NumericalSolutionOfDifferentialEquations"**

Homework		
Homework Set	Out	Due
5	26 Oct.	**16 Nov.**
6	**20 Nov.**	7 Dec.

Week of 17—21 November

Lectures		
	Topics	**Reading**
M 11/23 Lect. 22	Differential operators, damped and forced harmonic oscillators, non-homogeneous equations	*Kreyszig* **2.3,2.4, 2.7** (pages: 59–60, 61–69, 78–83)
W 11/25 Lect. 23	Resonance phenomena, higher order equations, beam theory	*Kreyszig* **2.8, 2.9, 3.1, 3.2, 3.3** (pages: 84–90, 91–96, 105–111, 111–115, 116–121)
F 11/27	Holiday, No Lectures	

Week of 30 Nov—4 December

Lectures		
	Topics	**Reading**
M 11/30 Lect. 24	Systems of differential equations, linearization, stable points, classification of stable points	*Kreyszig* **4.1, 4.2** (pages: 131–135, 136–139)
W 12/02 Lect. 25	Linear differential equations: phase plane analysis and visualization	*Kreyszig* **4.3, 4.4** (pages: 139–146, 147–150)
F 12/02	3.014 Meets, No Lecture	

Week of 6—11 December

3.014 Laboratory Week: 3.016 does not meet.

Homework		
Homework Set	Out	Due
6	20 Nov.	**7 Dec.**

Beginners to MATHEMATICA

Beginners to MATHEMATICA® tend to make the same kinds of mistakes. I've been collecting a list of such mistakes and present them to you as a reference tool.

I've constructed a few subjective usage paradigms and elements of style for MATHEMATICA® : http://pruffle.mit.edu/3.016-2009/mathematica-paradigms.html.

Lecture 01 MATHEMATICA® Example 1
Common Mathematica Mistakes
Download notebooks, pdf(color), pdf(bw), or html from http://pruffle.mit.edu/3.016-2008.

A list of common beginner MATHEMATICA mistakes. The entries here are typical **mistakes**. *I welcome input from others to might add to this list 1-7 are examples of confusing usages of parentheses (—), curlies {—}, and square brackets [—]. Generally, parentheses (—) are for logical grouping of subexpressions (i.e., (a+b)/(a-b)); curlies {—} are for forming lists or iteration-structures, single square brackets [—] contain the argument of a function (i.e., Sin[x]), double square brackets [[—]] pick out parts of an expression or list.*

1: *Probable error:* The parenthesis do not call a function, but would imply multiplication instead.

2: *Error:* The plot's range should be in curlies {—}.

3: *Error:* `Sort` should be called on a list, which must be formed with curlies—not parenthesis.

4: *Probable error:* If the intention was to multiply the list by a constant, then the first set of curlies turned the constant into a list, not a constant.

5: *Probable error:* If the intention was to extract the first element in the list, then double square brackets are needed (i.e., [[—]]).

6: *Error*: brackets cannot be used for grouping, use parentheses instead.

7: *Probable error:* The double brackets do not make a function call.

8–9: *Probable error:* MATHEMATICA® is case sensitive and functions are usually made by concatenating words with their first letters capitalized (e.g., `ArcCos`).

10: Functions are *usually* created designed with patterns (i.e., x_, y_) for variables. This is an error if `x` is a defined variable. This line is correct in using the appropriate *delayed assignment* :=.

12: *Probable error:* Here a function is defined with a direct assignment (=) and **not** *delayed assignment* :=. Because `x` was defined previously, the function will not use the current value of `x` in future calls, but the old one.

Forgetting or not understanding the differences between (···) {···} [···] [[···]]   **A**

(···) parenthesis are for logical grouping and are useful when you are unsure which operations take precedence--they have no operational meaning except for grouping parts of expressions. When in doubt about operator-precendence, use parenthesis: for example consider a/x+b--use a/(x+b) or (a/x) + b to avoid confusion

```
Cos (k x)
```
1

{···} curlies are for lists, iterators, and variable bounds
```
Plot[Sin[x], (x, 0, π)]
Table[Sin[x], (x, 0, 1, .1)]
```
2
```
Sort[(x, y, z)]
```
3

Multiplying objects of different types, or not using the intended type of multiplication
$$\left\{ \sqrt{2} \, \Big/ \, 2 \right\} \{a, b, c\}$$
```
{a, b, c} {1, 2, 3}
```
4

[···] single brackets are for function operations; double brackets pick out members of a list
```
SomeList = {a, b, c, d};
SomeList[1]
```
5
$$\left[(z^2 + y^2) \, c + b \, y^3 \right] a$$
6
```
Exp[[1]]
```
7

Forgetting *Mathematica's convention of capitalization of Functions*   **B**

Mathematic functions are a mixture of uppercase and lower case letters, the paradigm is that each word in a compound word is captialized, like this: ToBeOrNotToBe
```
arccos[1]
```
8
```
Arccos[1]
```
9

Forgetting to use patterns in function definitions (Underscores are Patterns)   **C**
```
MyFunction[x, y, z] := Sin[x] Sin[y] Sin[z]
```
10
$$MyFunction[\pi, \pi/2, 0]$$
11

Lecture 01　MATHEMATICA® Example 2

Common Mathematica Mistakes

Download notebooks, pdf(color), pdf(bw), or html from http://pruffle.mit.edu/3.016-2009.

(continued) list of common beginner MATHEMATICA® mistakes. The entries here are typical **mistakes** involving: the difference between assignment ($=$) and logical equality ($==$); forgetting commas; and, inadvertently reusing a defined variable.

Common Mistakes!

The difference between assignment = and equality testing ==

```
Solve[{h = 3 p + 4 q, k = 5 p - q}, {p, q}]
```
1

```
δ == 24
```
2

Not using enough commas

```
Plot[Sin[x + Exp[-x]] {x, 0, Pi}]
```
3

Forgetting that a variable has been defined

```
A = e^{-1.2/k 373}
```
4

Practical Advice 1:
Clear Variables

```
Clear[k];
A = e^{-1.2/k 373}
```
5

Practical Advice 2:
Second to last resort, clear everything

```
Clear["Global`*"];
```
6

Practical Advice 3:
Last resort, kill the kernel and restart it Use menu: Evaluation

1: *Probable error:* In the first line, assignments ($=$) are used instead of the double equals ($==$) which is a *logical equality*.

2: *Probable error* Assignment of the symbol δ was probably intended by here a logical equality is queried (i.e., Is δ the same as 24?) and no value is assigned to δ.

3: *Error* Commas separate arguments in functions like `Plot` that require at least two arguments.

4: *Probable error:* The symbol `k` was inadvertently assigned in the first line, and its value is still known to MATHEMATICA® .

5: Practical advice is to clear the variable definitions with `Clear`.

6: More powerful practical advice, but slight overkill, is to clear all user-defined variables. As a last resort when everything seems awry, kill the kernel with the menu and restart it. This starts up a new MATHEMATICA® session, but does not destroy the text in the `Notebook`.

Lecture 01 MATHEMATICA® Example 3

Common Mathematica Mistakes

Download notebooks, pdf(color), pdf(bw), or html from http://pruffle.mit.edu/3.016-2009.

(continued) list of common beginner MATHEMATICA® mistakes. The entries here are typical **mistakes** associated with using functions that are defined in Packages; assigning a variable to a formatted expression; and, using Evaluate within Plot.

Common Mistakes!

```
Histogram3D[
  Table[{RandomReal[], RandomReal[]}, {5}]]
```
1

```
<< Histogram`
```
2

```
SetDirectory[$InstallationDirectory]
FileNames[]
```
3

```
SetDirectory["AddOns"]
FileNames[]
```
4

```
SetDirectory["Packages"]
FileNames[]
```
5

```
<< Histograms`
```
6

```
Histogram3D[
  Table[{RandomReal[], RandomReal[]}, {20}]]
```
7

Mistake: Using formatting commands in assignments

```
mymat = {{1, 3, 7},
         {3, 2, 4},
         {7, 4, 3}} // MatrixForm
```
8

```
mymat = { {1, 3, 7},
          {3, 2, 4},
          {7, 4, 3}};
mymat // MatrixForm
```
9

```
Eigenvalues[mymat] // N
```
10

Not using Evaluate : slow and monochrome.

```
Plot[Table[LegendreP[i, z], {i, 1, 11, 2}],
  {z, -1, 1}, PlotStyle → Thickness[0.01]]
```
11

Using Evaluate : Fast and multicolored.

```
Plot[Evaluate[
  Table[LegendreP[i, z], {i, 1, 11, 2}]],
  {z, -1, 1}, PlotStyle → Thickness[0.01]]
```
12

1: Some of the less-used MATHEMATICA® functions are defined in *Mathematica Packages* and are not loaded automatically when MATHEMATICA® is started.

2: *Probable error:* Histogram3D is defined in Histograms

3–6 Demonstrate a method to find the names of the installed MATHEMATICA® packages. The current version of MATHEMATICA® 's help-browser (6.0) doesn't provide a way to find them. *It is probably a good idea to* Clear *the definition of a function like* Histogram3D *if you use it before loading its package.* Clear *before reading in the package.*

8: *Probable error:* here the formatting becomes part of the variable assignment. In this case, a MatrixForm of a matrix *is not* a matrix and so matrix operations are not defined (i.e., EigenValues would not produce the expected result).

9: Practical advice is to separate the definition from the display of the assigned variable. Here a matrix is defined; its MatrixForm is display, and Eigenvalues of the matrix can be calculated.

10: Some functions, such as Plot, evaluate their arguments in a round-about way. This produced an error in pre-6.0 versions of MATHEMATICA® . However, in 6.0, not using Evaluate makes the time of the computation long and will not produce a nice multicolored set of curves.

11: If a computationally intensive function is not doing what you expect, then try to wrap an expression in an Evaluate function—in this case it will tell Plot that it is operating on a list of particular size and produce a different color for each curve.