1. Use Mathematica to calculate an expression for Twinkies, where

\[ \text{Twinkies} = \int \frac{1}{\sqrt{1 + 2x^2 + x^4}} \, dx \]

Try to simplify the form of the expression. Does the expression seem as simple as it should be?

### 1

\[ \text{Twinkies} = \int \frac{1}{\sqrt{1 + 2x^2 + x^4}} \, dx \]

\[ \sqrt{\frac{1}{(1 + x^2)^2}} \, (1 + x^2) \, \text{ArcTan}[x] \]

Interesting. On first glance, it seems like the \((1+x^2)\) terms should cancel. However, I imagine that they don't cancel because Mathematica doesn't know anything about \(x\). Canceling the \((1+x^2)\) terms would require some assumptions about \(x\), which Mathematica won't make without me telling it to.

Now, we're told to simplify the expression. I don't actually expect this to get much simpler, but let's see what we get.

### 2

\[ \text{Simplify}[	ext{Twinkies}] \]

\[ \sqrt{\frac{1}{(1 + x^2)^2}} \, (1 + x^2) \, \text{ArcTan}[x] \]

That didn't get any simpler. Let's try FullSimplify[].

### 3

\[ \text{FullSimplify}[	ext{Twinkies}] \]

\[ \sqrt{\frac{1}{(1 + x^2)^2}} \, (1 + x^2) \, \text{ArcTan}[x] \]

That didn't get any simpler. So, this is evidently as simple as Mathematica can make it without making any assumptions about \(x\). I'm actually surprised that it's as simple as it is.

2. Verify that your expression is correct by taking the derivative of Twinkies.

### 4

\[ \text{D}[	ext{Twinkies}, x] \]

\[ \sqrt{\frac{1}{(1 + x^2)^2}} + \frac{2}{\sqrt{(1 + x^2)^2}} \, \text{ArcTan}[x] - \frac{2 \sqrt{\frac{1}{(1 + x^2)^2}} \, (1 + x^2)}{2 \sqrt{(1 + x^2)^2}} \]

This is not obviously the same as Twinkies. Let's try to simplify it to see if we can make it look like the integrand of
While this expression is not identical to the integrand in Twinkies, it IS obviously equal to the integrand in Twinkies. So, I’m satisfied that the integral is correct.

3. Use Simplify and the additional assumption that $x$ is a real number ($x \in \text{Reals}$). Name this result Twinkies.

$$\text{Twinkies} = \text{Simplify}[\text{D[Twinkies, }x\text{]}, \text{Assumptions} \rightarrow x \in \text{Reals}]$$

Here we added the assumption that Mathematica needed in order to cancel the $(1+x^2)$ terms. Now they’re gone and we have a very simple expression.

4. By integrating over the finite domain $x \in (0,y)$ where $y>0$, show that

$$\text{DingDongs} = \int_{0}^{y} \sqrt{\frac{1}{1 + 2x^2 + x^4}} \, dx$$

is the same function as the indefinite integral, Twinkies.

$$\text{DingDongs} = \int_{0}^{y} \sqrt{\frac{1}{1 + 2x^2 + x^4}} \, dx$$

Here we get the same result as we got in part 3, except that the variable is named $y$, rather than $x$. 
5. Plot DingDongs for $0 < y < 10$

\[
\text{Plot[DingDongs, \{y, 0, 10\}]}
\]

6. To generalize the above, consider the values of the definite integral over the variable domain:

\[ \text{JunkFood} = \int_{a}^{b} \sqrt{\frac{1}{a^4 + 2 a^2 x^2 + x^2}} \, dx \]

where $a > 0$, $b > 0$.

\[ \text{JunkFood} = \text{Integrate} \left[ \sqrt{\frac{1}{a^4 + 2 a^2 x^2 + x^2}} , \{x, 0, b\} , \text{Assumptions} \rightarrow a > 0 \& \& b > 0 \right] \]

\[ a^2 \text{ArcSinh} \left[ \frac{\sqrt{1 + 2 a^2 b}}{a^2} \right] \]

Here we get the integral evaluated both in terms of the variable $a$ and in terms of the integration limit, $b$. Note that this is no longer a function of $x$, because $x$'s "scope" is only within the integral. Thus, we end up with a function of two variables, $a$ and $b$. 
7. Plot the surface JunkFood for 0<a<3 and 0<b<3

Plot3D[JunkFood, {a, 0, 3}, {b, 0, 3}]

Here we get a 3D surface plot of our function of two variables.
8. Plot contours of constant value of JunkFood within $1 < a < 2$ and $1 < b < 2$

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ContourPlot[JunkFood, {a, 1, 2}, {b, 1, 2}]
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Here we have the contour plot over the appropriate domain. This is essentially the 2D projection of the 3D surface we plotted before, where lighter color indicates that the function has a higher value.