Problem Set 3: Due Friday 19 October, Before 5PM

Individual assignments should be a combination of your hand-worked solutions and other printed material—they should be placed in the mailbox outside Prof. Carter’s door. Email group assignments to 3016-psets(the symbol at)pruffle.mit.edu

For the individual problems so-indicated, you should work your solutions by hand and show your work. If not indicated, you may use and print the results of software-worked solutions.

The following are this week’s randomly assigned homework groups. The first member of the group is the “Homework Jefe” who will be in charge of setting up work meetings and have responsibility for turning in the group’s homework notebook. If some some reason, the first member in the list is incapacitated, recalcitrant, or otherwise unavailable, then the second member should take that position. Attention slackers: The Jefe should include a line at the top of your notebook listing the group members that participated in the notebook’s production; only those listed will receive credit. Group names are boldfaced text.

Fliberdigibbet: yulium, ghofmeis, bbaum, disko, wmr
Frateretto: breneman, mngarcia, mishu, aengwall, xavierg
Hoberdidance: mphurley, ktchang, agchen, bjornson, myananda
Tocobatto: ckubber, bonapart, chadtidd, kamo, leecm
Individual (Handworked) Exercise I3-1

*Kreyszig Problem Set 8.4* (problem 6, page 355)

Diagonalize:

\[
\begin{pmatrix}
2 & 7 \\
6 & -9
\end{pmatrix}
\]

Individual (Handworked) Exercise I3-2

*Kreyszig Problem Set 9.7* (problem 22, page 409)
(slightly modified from Kreyszig's)

Find the gradient of

\[
\ln(x^2 + y^2)
\]

and evaluate the gradient on the surface of a cylinder with unit radius with its axis running along the z axis.

Individual (Handworked) Exercise I3-3

*Kreyszig Problem Set 9.9* (problem 16, page 414)
(slightly modified from Kreyszig's)

a) Find the laplacian of

\[
f = z - 4\sqrt{x^2 + y^2}
\]

and convert the expression to cylindrical coordinates.

b) Find the same Laplacian, but first convert \(f\) to cylindrical coordinates, and then use the Laplacian operator in cylindrical coordinates.

Individual Exercise I3-4

*Kreyszig Problem Set 8.4* (problem 8, page 355)

Diagonalize:

\[
\begin{pmatrix}
-6 & -6 & 10 \\
-5 & -5 & 5 \\
-9 & -9 & 13
\end{pmatrix}
\]

Individual Exercise I3-5

*Kreyszig Problem Set 9.6* (problem 10, page 403)

Let \(w = f(x, y, z)\) and \(z = g(x, y)\), find all partial derivative of the scalar field \(w\) constrained to the graph \(z\).

Work it out for the particular case of \(w = f(x, y, z) = x^3 + y^3 + z^2\) and \(z = g(x, y) = x^2 + y^2\).

Individual Exercise I3-6

Determine if \( \cos(xyz)(yz, zx, xy) \) has a scalar potential and, if so, find it.

**Individual Exercise I3-7**  
*Kreyszig Problem Set 9.8* (problem 20, page 414)  
Let \( w = f(x, y, z) \) and \( z = g(x, y) \), find all partial derivative of the scalar

**Individual Exercise I3-8**

Write a function that takes a vector of arbitrary length and returns a unit vector parallel to the original vector. Demonstrate that it works.

**Group Exercise G3-1**

Write a function that visualizes the tangent vector, the normal vector, and the approximating circle (called the osculating circle) for each point along the curve:

\[
\vec{x}(t) = ((2 + \sin 2t) \cos t, (2 + \cos 2t) \sin t, \cos t + \sin t)
\]

**Group Exercise G3-2**

The goal of this exercise is to write a “seemingly intelligent Professor” function. The idea is to create “Mad-Libs” that give an answer to a student’s classroom question, when the Professor clearly doesn’t know the answer.

For example, you can compile lists of introductory clauses, prepositions, verbs, objects of a preposition, etc., that get concatenated and return a random, but grammatically correct, sentence.

As a bonus, you should write this as a function of a single “stupidity” argument, \( S \in (0, 1) \), where \( S = 0 \) is a brilliant question and \( S = 1 \) is kind of question that a Harvard student might ask. For example, you might rank your phrase-lists so that there is an increased the probability that a “polite” answer is given to a brilliant question, and a “snarky” answer for Harvard-like questions.

**Group Exercise G3-3**

The objective of this problem is to explore and visualize how a distribution of states will evolve if state transitions are governed by local changes in energy. Students should decide what are the important *generic* physics of such an evolution and steady-states as a function of relevant parameters and present an exposition that illustrates those physics. The problem statement is purposely vague—the best solutions include elements of design and creativity as well as physical understanding.

For example, consider the following “energy landscape” where the vertical axis is potential energy as a function over the plane (for example \( E = mgh(x, y) \) or \( E = qV(x, y) \)): 
Suppose there is an initial distribution of states (i.e., $\rho(x, y, t = 0) dx dy$ is the number of states in a square with area $dx dy$ with its center at $(x, y)$ at time $t = 0$).

Suppose that each state at $\vec{s} = (x, y)$ randomly samples its neighborhood and makes a transition to a new state $\vec{s}' = (x + \Delta x, y + \Delta y)$ with the following probability (known as the Metropolis algorithm):

$$\text{probability of transition} = \begin{cases} 1 & \text{if } E(\vec{s}') \leq E(\vec{s}) \\ \exp \left[ -\frac{E(\vec{s}') - E(\vec{s})}{kT} \right] & \text{if } E(\vec{s}') > E(\vec{s}) \end{cases}$$

where $T$ is the temperature.

Suppose that all states start in the neighborhood of $(x = 0, y = 0)$, and iterate the transitions to see how the distribution changes with the number of steps.

You will need to construct your own function that (at least) has the properties illustrated in

You will want to pick a “effective temperature” so that $kT$ is on the order of the difference between a minimum and a saddle point.

Groups should develop several methods of visualization under different illustrative conditions and collect them together into a coherent exposition of such activated evolution.

Students may ask for clarification to the problem statement and receive hints, but no hints will be given about the “best way to visualize” or “what physical principles should be displayed.” These are left to your own curiosity and creativity.