Lecture 2: Introduction to Mathematica

Expressions and Evaluation

There are very many ways to learn how to use MATHEMATICA®. Nearly all of the best ways involve performing examples from the very beginning. That is how we are going to start—with examples. Using MATHEMATICA®’s FrontEnd you may execute a command by pressing Shift-Enter; simply pressing Enter tells MATHEMATICA®’s that you merely wish to have a “carriage return” on the screen.

Mathematica’s syntax will feel fairly natural after a while. Use the following notebook to get started. Execute a few commands until you get a sense for what output MATHEMATICA® will produce; try editing the commands; try to make MATHEMATICA® do something strange—just try playing with it and you will soon get the hang of what is going on.

One way to use MATHEMATICA® is simply as a calculator that allows symbols to get carried along. MATHEMATICA® will usually try to resolve every symbol and return precise information about it. If something is undefined to MATHEMATICA®, it simply returns it as a symbolic expression.

A number is not returned until all of the symbols in an expression are defined as numbers. MATHEMATICA® will try to be exact—it does not calculate $\frac{1}{3} + \frac{1}{2}$ by adding $0.33333\cdots + 0.5 = 0.83333\ldots$, it has an algorithm for adding rational numbers and gives $\frac{5}{6}$. 
Getting Started

There are a variety of ways to get Mathematica® started and these are specific to the operating system your computer uses. A license must be purchased to run Mathematica® code, but free Mathematica®-display tools can be obtained from Wolfram.

The FrontEnd is the graphical interface between the user and Mathematica®—you arrange your Mathematica® input, sometimes with text-like comments, in the FrontEnd. The user must request the FrontEnd to pass something to Mathematica®’s kernel, by pressing Shift-Enter. The kernel is the resident symbolic algebra software engine behind Mathematica®.

The appearance of the FrontEnd depends on either provided or user-designed StyleSheets. The StyleSheet for this course can be downloaded from the course website. The course style is particularly ugly—it is hoped that this will provide an incentive for students to create their own style.
Basic Input and Assignment

The methods of assigning expressions (expr) to symbolic variables (SomeVariable) via expr = SomeVariable. Differences between exact (symbolic) objects and numerical objects. Logical equalities (==) and Clearing symbols. Many bugs crawl into MATHEMATICA® from “uncleared” symbols.

1: A symbol is assigned to an expression with an equals sign =. Some symbols, such as π, are already defined—in MATHEMATICA® it is exactly the ratio of a circle’s circumference to its diameter.

2: If a previously assigned symbol is used in a new assignment, MATHEMATICA® will usually incorporate the previously assigned symbol’s properties. Here, the variable is given a descriptive and long name: this is good practice if you want your code to self-document. Your definitions can be recalled and used to build up more complicated expressions.

6: It is possible to assign symbols to numerical objects; here b is a different kind of object (numerical) than the previously defined a (symbolic). There are built-in object-types created from combinations of other types.

10: MATHEMATICA® will to be exact—the ArcCos of −1 is exactly π.

11: However, the ArcCos of −1.000 is a numerical approximation to π—the result will be numerical (not exact).

13: A Rational expression is another type of MATHEMATICA® object. There are built-in rules to treat rational expressions exactly. The result of applying N to the rational 5/6 is a numerical approximation to 5/6.

14: Assignment = is different from logical equality ==. For a logical equality, the result is either True, False, or MATHEMATICA® cannot determine and returns an expression.

17: It is possible to clear all user-made definitions. A last resort would be to instruct the FrontEnd to kill MATHEMATICA®’s kernel and restart it. clearing symbols.
Mathematica® has many, very many, built-in mathematical functions; it’s probably impossible to memorize all of them. You can usually find what you need by using the Help Browser, or by querying \? with a wildcard * such as in \?*Geom*.

1: Mathematica® has a fairly consistent function naming strategy. The first letter of a word is always capitalized; compound words are concatenated together while maintaining the first letter capitalization; thus `InverseBetaRegularized`. A function is just another symbol—if a symbol is followed by square brackets [], the stuff inside the brackets become the argument(s) for the function.

4: There are usually more than one way to do things. The operator // is a short-hand way of applying the function that follows // to the expression that proceeds it (e.g., \(\pi/2\)//Sin). You can also use @ to prefix a function (e.g. Sin@(\(\pi/2\))).

6: There are methods designed to improve or alter the appearance of complicated expressions.

8: Spelling errors can be very difficult to spot and debug—as a matter of practical advice, look for Mathematica®’s spelling warnings. They can be turned off with Off[General::spell], but it is not generally a good idea.

9: Mathematica® has a sophisticated symbolic integration algorithm...

10: even if sometimes it expresses it in special functions
The derivative and integration methods are introduced. Simple plotting methods are demonstrated with an example of annotating a plot.

2: If Mathematica® can’t differentiate or integrate a function, it will be left in a symbolic form.

5: Mathematica® applies the fundamental theorems of calculus...

10: The calculus operations will often create long and complicated expressions. That two expressions are equivalent can sometimes be shown with built-in functions such as Simplify, FullSimplify, Factor, Expand, Collect, etc., but sometimes it is an art to turn an expression into an aesthetic form.

12: This is the simplest form of Plot. The second argument is a list giving the variable and its bounds. The first argument should have a numerical value at most of the points within the variable’s bounds.

15: To find all the possible options for a function with their default values, the Options function provides a way to decipher what aspects of a plot can be changed easily. Demonstration of a function that Mathematica® does not know how to integrate or differentiate.

16: Here is an example with a plot title, axes labels, different colors and thickness for the curves.
Lists, Lists of Lists, and Operations on Lists

Lists are useful ways to keep related information together, and Mathematica® uses them extensively. Lists could be created in Mathematica® by using the List function, but they are usually entered in with curly-brackets {}.

3: Some functions, such as Cos here, are *threadable functions*; when called on a list-argument, they will produce a list of that function applied to each list element.

4: A list’s parts (or, elements) can be picked out in a variety of ways. The Part function has a shorthand double-bracket form.

7: There are plenty of functions designed to operate on lists.

8: Logical operations, such as NumberQ, can be used to select elements by their characteristics.

13: A list’s elements can be lists themselves. For example, a matrix is represented by list of a list. And their are higher-dimensional structures such as tensors. Dimensions is a useful way of learning about such structures.

14: Here, the post-fix operator for a function is used to change the way a matrix is displayed. Note, the result is not a matrix, but a DisplayForm of a matrix.

22: This is a fairly advanced example of extracting the odd-numbered columns of a matrix. The list IntList is simply the integers for each column; its odd-numbered members are selected and become the second (i.e., column) argument of the Part selection. The first argument is All, so the entire row is captured for each selected column.
A rule \texttt{leftvar} $\rightarrow$ \texttt{rightvar} is similar to assignment in that it associates a new symbol (\texttt{leftvar}) with something else, but it the value is not assigned—it does not effect future values of the left-hand-side symbol. Rules are often used in conjunction with replacements. Many of \textsc{Mathematica}® functions, (e.g., \texttt{Solve}) return rules as a result.

1: The rule $a \rightarrow \pi/3$ is assigned to the symbol \texttt{ARule}.

3: A rule can be applied with the function \texttt{Replace}, but the syntax (.) is typically used instead.

4: Rules can be collected into lists.

5: Assignment of $a$ is reflected in the form of \texttt{ARule}.

8: Rules are necessary for manipulations in \textsc{Mathematica}®, but can be used to generate “mistakes.” Think of Rule and Replace acting on an expression as “What would the expression be if a certain rule were applied to it?” If the rule is wrong, the resulting expression will be as well.
Getting Help on Mathematica

MATHEMATICA®’s built-in help functions are very useful. This was true even before the whole MATHEMATICA® manual was incorporated into the Help Browser. In the old days, one would memorize large portions of the MATHEMATICA® book—which has grown continuously heavier since its first publication in the early 1990’s—and rely on the useful ”?” and ”??” operators. The use of ”?” with the wildcard ”” enabled a beginning user to track down almost any MATHEMATICA® function. The Options function is also a very efficient way to discover alternative ways of getting results.

I would have recommended ‘scanning’ the entire MATHEMATICA® manual in a single three hour sitting (about 600 pages per hour) as an effective way to acquire a working familiarity with the software, but I don’t because the built-in browser is so easy to use.

I encourage you to idly explore the MATHEMATICA® Help Browser. You will not only learn about MATHEMATICA®, but also about mathematics.
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