Overview

This laboratory provides an exercise in the solution to an ODE by a direct and a numerical method.

Tasks

Radial Steady-State Diffusion across a Cylindrical Membrane

The objective is to find the steady-state temperature distribution (i.e., steady-state implies that $\partial T/\partial t = 0$) in a cylindrical wall where the inner and outer temperatures are fixed at constant values.

Suppose that the temperature, T is a function of only the radial coordinate, r (i.e., $T(r)$). The steady-state diffusion equation is

$$\nabla \cdot D\nabla T(r) = 0 \tag{8-1}$$

where D is the thermal diffusivity. The boundary conditions at the inner radius, r_{in}, and the outer radius, r_{out}, are

$$T(r_{in}) = T_{in} \qquad \text{and} \qquad T(r_{out}) = T_{out} \tag{8-2}$$

1. For the case the D is constant and uniform, find the solution, $T(r)$, to the steady-state profile.
   Hint: You may wish to load the `Calculus‘VectorAnalysis‘` package that has `Div`, `Grad`, and `Laplacian` in several coordinate systems.
   Plot the temperature distribution $T(r)$ for the case that $r_{in} = 1$, $r_{out} = 2$, $T(r_{in}) = 1$, $T(r_{out}) = 2$, and $D = 0.0186579$.
2. Suppose that the thermal diffusivity decreases very rapidly with radial coordinate $D(r) = \exp[-\exp(r)]$. Can MATHEMATICA® find a symbolic expression for the steady-state solution?
3. Find a numerical solution for $T(r)$ for the above case ($r_{in} = 1$, $r_{out} = 2$, $T(r_{in}) = 1$, $T(r_{out}) = 2$, and $D(r) = \exp[-\exp(r)]$). Plot the temperature distribution.

Report

This homework will be graded. Your report on the work above should be ordered as it is above. Your report should include comments that would help one of your classmates understand what your work demonstrates. Send your report as a saved Mathematica notebook with name `3016_Lastname_Lab08.nb` to 3016-labreports@pruffle.mit.edu; as the subject use "3.016 Lab 08 LASTNAME".