Overview

This laboratory should give students experience with some of the basic methods for solving matrix equations.

Tasks

Solving Systems of Linear Equations

In linear elastic materials, it is convenient and standard to write the elements of the stress tensor, σ_{ij}, and the elements of the strain tensor, ϵ_{kl}, as entries in vectors of length 6:

$$\vec{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{pmatrix} \quad \text{and} \quad \vec{\epsilon} = \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ 2\epsilon_{yz} \\ 2\epsilon_{zx} \\ 2\epsilon_{xy} \end{pmatrix}$$

In this vector notation, the stresses and strains are connected through a 6×6 *compliance matrix* $\underline{S}$ as follows:

$$\vec{\epsilon} = \underline{S}\vec{\sigma}$$

The compliance matrix will inherit the symmetry of the material. For a tetragonal material with point group symmetry *4mm*, there are only six independent elements of the compliance matrix. Every *4mm* material's compliance matrix will look like this:

$$\underline{S} = \begin{pmatrix} s_{xxxx} & s_{xxyy} & s_{xxzz} & 0 & 0 & 0 \\ s_{xxyy} & s_{xxxx} & s_{xxzz} & 0 & 0 & 0 \\ s_{xxzz} & s_{xxzz} & s_{zzzz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 4s_{yzyz} & 0 & 0 \\ 0 & 0 & 0 & 0 & 4s_{yzyz} & 0 \\ 0 & 0 & 0 & 0 & 0 & 4s_{xyxy} \end{pmatrix}$$

1. For a uniform shear-free dilation that changes the relative volume by a factor $\Delta V/V$, the strain vector is given by

$$\vec{\epsilon}_{uni} = \frac{\Delta V}{3V} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

   Find an expression for stresses that would produce a uniform shear-free dilation in a single crystal with *4mm* symmetry.

2. Find additional conditions on the s_{ijkl} such that the stress that gives rise to a uniform shear-free dilation is a hydrostatic pressure, P, i.e.,

$$\vec{\sigma}_{hyd} = -P \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

3. Suppose you could do experiments in which you set (in other words, specify) the stress, $\vec{\sigma}$, and measure the strain, $\vec{\epsilon}$. How many different experiments would you need to do in order to determine the compliance matrix for a *4mm* single crystal?
4. Justify your answer above by specifying the experiments which would be done and show that they would determine $\underline{S}$ for *4mm* symmetry.

Save your Work Save your work as a mathematica notebook: `3016_Lastname_Lab03.nb`.

Report

This homework will be graded. Your report on the work above should be ordered as it is above. Your report should include comments that would help one of your classmates understand what your work demonstrates. Send your report as a saved Mathematica notebook with name `3016_Lastname_Lab03.nb` to 3016-labreports@pruffle.mit.edu. As the subject use "3.016 Lab 03 LASTNAME".