Massachusetts Institute of Technology

Mathematical Methods for Materials Scientists and Engineers

3.016 Fall 2006

W. Craig Carter

Department of Materials Science and Engineering
Massachusetts Institute of Technology
77 Massachusetts Ave.
Cambridge, MA 02139

Problem Set 5: Due Wednesday 13 Dec, Before 7PM

Individual assigments should be a combination of your hand-worked solutions and other printed material—they should be placed in the mailbox outside Prof. Carter's door. Email group assignments to mingtang(the symbol at)mit.edu

For the individual problems so-indicated, you should work your solutions by hand and show your work. If not indicated, you may use and print the results of software-worked solutions.

The following are this sets's assigned homework groups. The first member of the group is the "Homework Jefe" who will be in charge of setting up work meetings and have responsibility for turning in the group's homework notebook. If some some reason, the first member in the list is incapacitated, recalcitrant, or otherwise unavailable, then the second member should take that position. *Attention slackers:* The Jefe should include a line at the top of your notebook listing the group members that participated in the notebook's production; only those listed will receive credit. Group names are boldfaced text.

Bull Moose: *lhunting, cmurphy, soyegg, barnest, ajuneau, samok*

Carpetbaggers: *pantea, scampini, dian88 , zkal64, s_evans, saz*

Muckrackers: *buchok, k_chu, ielaine, aishab, sadik, m_scot*

Suffragettes: *brasin, katpak, jskrones, rachelkl, jszab, davidlin*

Scalawags: *rkusko, tejada, tycosknr, raffael, alanho12, zaklouta*

Wobblies: *cgh, johannk, arpun, siamrut, avadhany, taf*

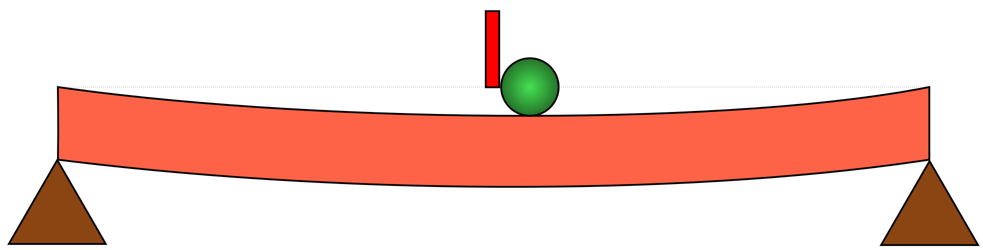
Individual (Handworked) Exercise I5-1

Kreyszig **Problem Set 2.4** (problem 14, page 69) Consider an underdamped motion of a body with mass $m = 2$ kg. The time between two consectutive maxima os 2 sec and the maximum amplitude is reduced to 1/4 of its initial value after 15 cycles, calculate the damping constant for this system.

Individual Exercise I5-2

The objective is to roll the ball with density ρ across a beam with thickness w, length L, and elastic modulus E. Gravity is acting downwards with a constant given by g.

The path is blocked by a rigid obstacle at the same height as the unloaded beam. Treat the beam and loading as a linear elastic problem (and while it is redundant to state this, ignore the density of the beam).

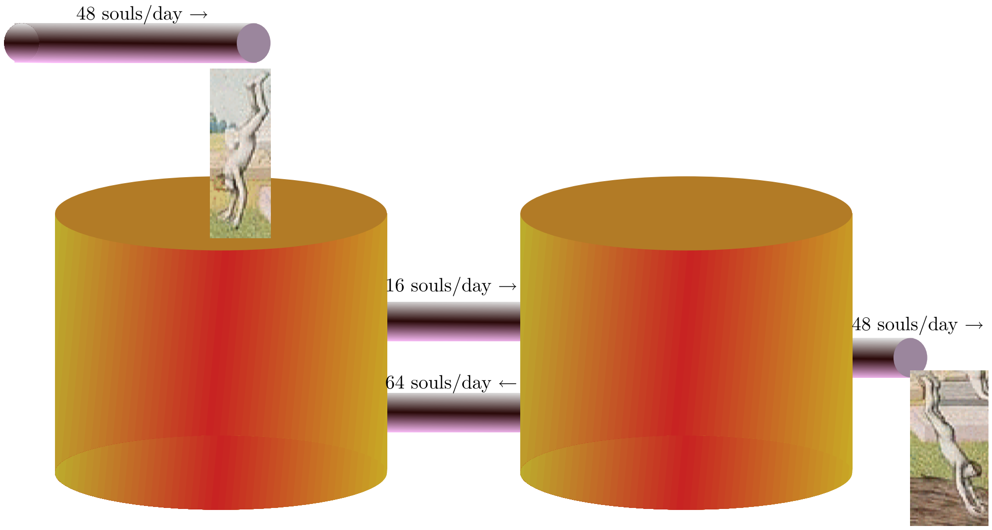


Calculate the conditions that the ball can roll past the obstacle.

Individual Exercise I5-3

Variation on *Kreyszig* **Problem Set 4.3** (problem 18, page 146) (*Kreyszig* **Problem Set 3.3** (problem 16, page 169) in 8th edition)

Last September, first two levels of the inferno were filled with 400 souls; the first level was populated with the souls of of 100 MIT students and the second level was populated with 40 MIT-student souls. Souls were demoted as indicated in the following picture:



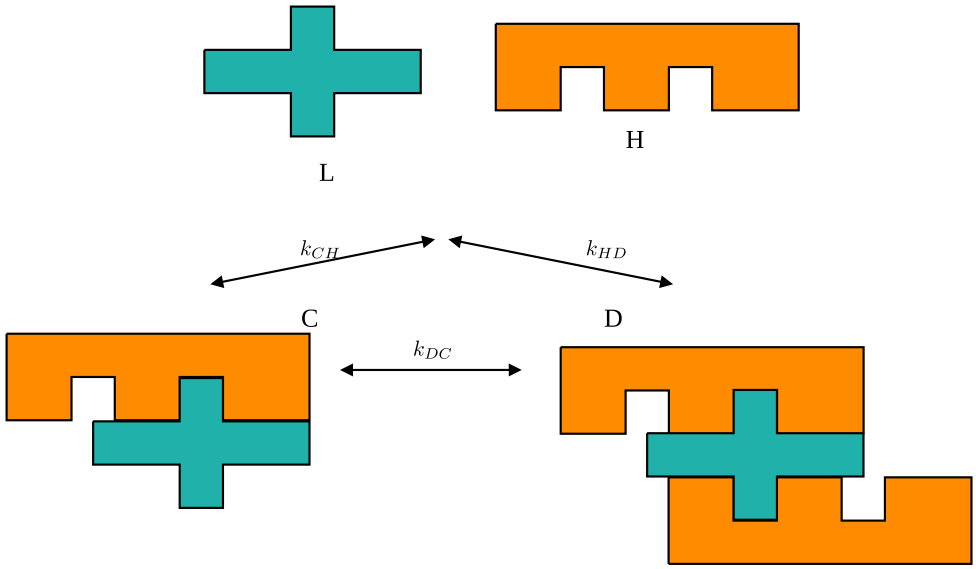
Calculate and plot the number of MIT students in each level as a function of time for three cases: 1) the souls entering the top level are all Harvard students. 2) the souls entering the top level are 50% MIT students and 50% Harvard students. 3) the fraction of MIT souls entering goes like $0.5(1 + \sin(2\pi t/56))$ (t in weeks)

extra credit Amusing variations on this theme are welcome.

Group Exercise G5-1

A kinetic model for the development of complexes in human growth hormone (hGH) was developed by J.M. Haugh, *Biotechnology Prog.*, 1337–1344, **2004**. The goal is to analyze a simplified version of this model.

In this simplified model, ligands L combine with one of two empty receptors R in hGH to form a complex C. The complex, C, can either form a dimer, D, by combining with another empty R or release it ligand return to its R-state. The dimer can lose an L to reform a complex C.



The model for this process is:

$$\frac{dH}{dt} = \dot{\chi}_H + k_{CH}C + k_{HD}D - (k_{CH}L + k_{DC}C)H$$
$$\frac{dC}{dt} = k_{CH}LH + k_{DC}D - (k_{CH} + k_{DC}C)H$$
$$\frac{dD}{dt} = k_{DC}CH - (k_{DC} + k_{HD})D$$

where in the above equation, H is the concentration of un-blocked hGH, C is the concentration of ligand-hGH complexes, and D is the number of hGH-ligand-hGH dimers. L is the ligand concentration and $\dot{\chi}_H$ is the rate of production of hGH per unit volume.

This model has five free parameters. For example, scale the time-variable to reduce the number of parameters to four: a normalized hGH production rate, two independent ratios of kinetic coefficients k_i, and the ligand concentration L.

Find the fixed-point (possibly steady-state) composition for this system a function of the free parameters.

Set $k_{CH} = k_{DC} = k_{HD}$ and use the L–$\dot{\chi}_H$ plane to illustrate the characteristics of the fixed point.

Group Exercise G5-2

This is an unusual quantum mechanics problem, but it should be a legitimate model.

The potential for a object of mass m in a constant gravitational field is

$$V(z) = mgz$$

Find the general solution to the one-dimensional time-independent Schrödinger equation

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(z)}{dz^2} + V(z)\psi(z) = E\psi(z)$$

where $\psi(z \leq 0) = 0$ for the first four lowest energies.

You will have to find the zeroes of and normalize functions of the Airy function. You may need to resort to numerical methods.

Plot the probability density of the wave-function $\bar{\psi}\psi$ for these three energy levels.