Massachusetts Institute of Technology

Mathematical Methods for Materials Scientists and Engineers

3.016 Fall 2006

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Problem Set 2: Due Friday 13 October, Before 5PM

Individual assigments should be a combination of your hand-worked solutions and other printed material—they should be placed in the mailbox outside Prof. Carter's door. Email group assignments to mingtang(the symbol at)mit.edu

For the individual problems so-indicated, you should work your solutions by hand and show your work. If not indicated, you may use and print the results of software-worked solutions.

The following are this week's randomly assigned homework groups. The first member of the group is the "Homework Jefe" who will be in charge of setting up work meetings and have responsibility for turning in the group's homework notebook. If some some reason, the first member in the list is incapacitated, recalcitrant, or otherwise unavailable, then the second member should take that position. *Attention slackers:* The Jefe should include a line at the top of your notebook listing the group members that participated in the notebook's production; only those listed will receive credit. Group names are boldfaced text.

Adiantum: *samok, cgh, brasin, arpun, taf, rkusko*

Asphodelus: *s_evans, lhunting, cmurphy, xsu, zkal64, soyegg*

Carduus: *jskrones, pantea, johannk, debralin, zaklouta, buchok*

Corylus: *katpak, m_scot, dian88, tycosknr, alanho12*

Delphinium: *k_chu, aishab, ielaine, scampini, ajuneau*

Ligustrum: *rachelkl, davidlin, raffael, saz, jszab*

Salvia: *barnest, siamrut, cgh, tejada, sadik*

Individual (Handworked) Exercise I2-1

Kreyszig **Problem Set 7.3** (problem 16, page 295) Solve:

$$\begin{array}{lllll} -2w & -17x & +4y & +3z & = 0 \\ 7w & & +3y & -2z & = 0 \\ & 2x & +8y & -6z & = -20 \\ 5w & -13x & -y & +5z & = 16 \end{array}$$

Individual (Handworked) Exercise I2-2

Kreyszig **Problem Set 8.1** (problem 16, page 339) Find eigenvalues and eigenvectors of:

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{5} & \frac{1}{10} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{7}{2} \end{pmatrix}$$

Individual (Handworked) Exercise I2-3

Kreyszig **Problem Set 8.4** (problem 8, page 355) Diagonalize:

$$\begin{pmatrix} -6 & -6 & 10 \\ -5 & -5 & 5 \\ -9 & -9 & 13 \end{pmatrix}$$

Individual Exercise I2-4

Kreyszig MATHEMATICA® *Computer Guide:* problem 6.7, page 77

Individual Exercise I2-5

Kreyszig MATHEMATICA® *Computer Guide:* problem 6.14, page 78

Individual Exercise I2-6

Kreyszig MATHEMATICA® *Computer Guide:* problem 7.8, page 87

Individual Exercise I2-7

Kreyszig MATHEMATICA® *Computer Guide:* problem 7.10, page 87

Individual Exercise I2-8

Write a program that fills a 6×6 matrix with random integers from a uniform distribution of $(-6, -5, \ldots, 5, 6)$. Find a way to visualize:

1. The probability that the determinant is zero.
2. The probability of finding an eigenvalues of a given magnitude.
3. The probability of finding a particular eigenvalue.

How do these change if the random integers are chosen from $(1, 2, 3, 4, 5, 6, 7)$?

Group Exercise G2-1

In two dimensions there are a set of symmetry operations on points $\vec{v}$:

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

that can be represented by matrix operations on vectors:

$$\underline{M}\vec{v} = \begin{pmatrix} m_{xx} & m_{xy} & mxz \\ m_{yx} & m_{yy} & myz \\ m_{zx} & m_{zy} & mzz \end{pmatrix}$$

Among the possible symmetry operation are:

Mirror Reflections across the x, y, and z-planes

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{and} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Rotation by θ about the x, y, and z-axes

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \text{and} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Use these operations and modify the MATHEMATICA® example in http://pruffle.mit.edu/3.016/ProblemSets/HW-2-Setup.nb and illustrate an object that has:

1. An object with mirror symmetry across the x-, and y-planes, and a four-fold rotation symmetry about the z-axis.

2. Four-fold symmetry about the x-axis, three-fold symmetry about the y-axis, two-fold symmetry about the z-axis.

You may wish to be careful that you do not introduce additional symmetries.

Group Exercise G2-2

The electrical conductivity, σ, is a second-rank tensor property that relates the current density vector, $\vec{J}$, to the electric field, $\vec{E}$ by

$$\vec{J} = \sigma \vec{E}$$

In a particular coordinate system, the electrical conductivity of a tetragonal tin single crystal was measured to be:

$$\sigma = \begin{pmatrix} 8.55 & 1.55 & 0 \\ 1.55 & 8.55 & 0 \\ 0 & 0 & 10.1 \end{pmatrix} \times 10^6 \quad (\mathrm{ohm} \cdot \mathrm{m})^{-1}$$

1. Find the current density vector and its magnitude when an electric field of 0.1 V/m is applied parallel to the "2" axis in this coordinate frame.

2. Find the electric field, $\vec{E}$, which will create a current density $\vec{J}$ of magnitude 10^{-6} coulomb $\mathrm{s}^{-1}\,\mathrm{m}^{-2}$ flowing *in the direction of* (111).

3. The vector

   $$\hat{E} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

   represents electric fields with unit magnitude directed towards θ in the x–y plane. The angle, α, between two vectors $\vec{J}$ and $\vec{E}$ is given by

   $$\cos\alpha = \frac{\vec{J} \cdot \vec{E}}{|\vec{E}|\,|\vec{J}|}$$

   Plot the angle between the current density and the electric field for for all the unit vectors, $\hat{E}$, given above. In other words, plot $\alpha(\theta)$.

4. Find the eigenvalues and eigenvectors of tin's electrical conductivity.

5. Demonstrate that, if an electric field is applied in the direction of one of the eigenvectors calculated above, the current density will be parallel to the electric field.

6. Visualize the quadratic forms representing tin's conductivity and resistivity each of their principle axes.