Sept. 07 2005: Lecture 1:

Introduction and Course Description

Preface

The subject is for undergraduate materials scientists and engineers who wish to learn about the mathematics that is essential to their chosen field.

Materials science and engineering is a discipline that combines knowledge of chemistry, mechanics, and physics and then applies them to the study of materials and their properties. It is a challenging and diverse enterprise—obtaining expertise in a large set of diverse subjects—but a career devoted to diverse study and applications will be very rewarding and fulfilling.

Mathematics is the *language* that binds together disparate topics in physics, engineering, and chemistry. While it is possible to become an excellent materials scientist and engineer without some working knowledge of a large subset of mathematical topics, it is much easier to master this discipline with mathematics to guide you. Through mathematics, you will discover that some topics have similarities that are not obvious—and not taught to you as *being* similar. Such similarities and analogies will make learning much, much easier—and I think much more enjoyable.

MIT's Department of Materials Science and Engineering has determined that students benefit from a background and a working knowledge of many more mathematics topics that pertain specifically to MS&E than are taught in a one-semester subject in the mathematics department. It is reasonable to ask, "Is this subject a substitute for a mathematics subject such as linear algebra or partial differential equations taught by a mathematics professor as part of the mathematics department?" No, this is not a replacement for such a subject and I encourage you to take subjects in mathematics in the future. This subject is designed to be very broad in scope and therefore its depth in any one topic is limited.

I do believe very strongly that you will enjoy studying math more after taking this introduction and that the mathematical background you will receive this semester will make your materials science education richer and more rewarding.

I have designed this subject to help you learn as much essential mathematics as possible in a short time. To this end, this subject has several unusual aspects that you will need to know.

3.016 Mathematical Software

Symbolic mathematical computer software is a tool used by almost every applied scientist. Such software helps produce results quickly, visualizes and documents the results, and minimizes the silly errors that creep into complicated mathematical manipulations. Although there are many other good choices, I have decided to use Mathematica® as a vehicle for learning and doing mathematics. It has a fairly steep learning curve, but it probably repays the time investment with powerful (once learned) language syntax and packages. Such symbolic mathematical software is an aid to help you think about and perform mathematics—it is not a replacement for mathematical understanding.

Mathematica® is available for all MIT students, both on Athena (free) and via licenses for personal laptop and desktop machines ($30 for students, a useful investment for other subjects). The process to access Mathematica® on Athena and the steps to download a license will be explained to you; the pertinent website is http://web.mit.edu/is/products/vsls/ http://web.mit.edu/is/products/vsls/. You will need Mathematica® for your first homework set and laboratory; you should try to get it working someplace very soon. If you have a laptop, I suggest that you install MATHEMATICA® on it as soon as possible.

Homeworks and lab reports will be turned in as MATHEMATICA® notebooks, by mailing to 3.016(a)pruffle.mit.edu.

Laboratory assignments must be completed during the laboratory period and are to be emailed as an electronic copy of a MATHEMATICA® notebook to the instructor and the TA.

3.016 Examination Philosphy

Tests and exams are powerful motivators to get students to take a subject seriously but I think that working through homework problems better promotes learning, particularly for self-motivated students.

Therefore, there will be no exams, tests, or quizzes in 3.016. Your entire grade will be based on your homeworks and laboratories. These will graded carefully (described below) and there will be about one homework set per week while the subject is meeting (i.e., no homework will be assigned during the weeks that the laboratory 3.014 meets).

3.016 Homework

Of course when you do homework, you are not under the potentially menacing eyes of an exam proctor. This means that you can receive help in the form of:

Books Go to the library and find solutions to problems. It is good practice and you will learn quite a bit by doing so. I recommend that you attempt to find a solution before going to the library—not only will it help you appreciate the solution, it will also make your search a bit easier!

Experts By all means, consult with experts on your homework. It is a good idea as long as you understand what *you* turn in.

Classmates This is the best choice of all. I think it is both inevitable and beneficial to give and receive help. Cooperating on homework will help you learn to communicate your ideas and begin to appreciate the difficulties and rewards of teamwork.

As explained below, the homework assignments in 3.016 will be, in part, cooperative.

You will find that you are more busy some weeks than others and relying on a classmate during a busy week can be a life-saver. However, if you start slacking off and don't hold up your end of the bargain when you are able, you will engender resentment and endanger professional and friendly relationships. I leave it to your own conscience to play fairly and contribute when you can—and, while understanding that everyone experiences different kinds of pressures, to be forthright and honest with others who do not contribute consistently.

Homework cooperation has a potential downside because you all receive individual grades. We will attempt to mitigate this downside by dividing the homework into two parts:

Group For each homework set, a few problems will be designated as *Group Exercises*. For these problems, the entire group will turn in *one homework*. Every member of the group will receive exactly the same credit for the homework grade.

Homework groups will be *assigned* with each homework set. The groups will change from week to week and the members will be assigned randomly. Each group will be assigned a homework leader who will be responsible for arranging meetings and turning in the homework.

Individual Each problem set will contain a few problems for each student to complete individually. These problems will come out of the textbook and tend to be a bit easier than the group exercises. They are designed to maintain a sufficient amount of currency and emphasize that reading the textbook is an essential part of this course.

3.016 Laboratory

There will be a laboratory each week that 3.016 meets. The labs will be practical and focussed on using MATHEMATICA® effectively.

There will be assigned reading from the MATHEMATICA® help browser that comes with the software. *You should always do this reading before the laboratory, or you may not be able to* FINISH YOUR ASSIGNMENT AND TURN IT IN BEFORE THE END OF THE LABORATORY IN ORDER TO GET CREDIT.

If you stay current in the course material and do the homeworks, you should have no difficulty doing the laboratory assignments if you do the pre-assigned reading.

It is not necessary, but if if you have your own laptop running MATHEMATICA® , it will be helpful to bring it to the lab with you.

Grades

As stated above, all of the final grade will depend on the homeworks and the laboratory assignments. There is no fixed average grade for this course; the average will depend on the entire class performance. However, if your homework grades and your laboratory reports are consistently within the top quartile, then it is extremely likely that you will receive an **A**. Homeworks will be graded by ranking them in order from *Best Homework* to *Least Best Homework*. A decision will be made regarding how many points (out of a possible 100) the *Least Best Homework* deserves, and the homework scores will be interpolated between a score of 99 for the *Best Homework* and that of the *Least Best*.

Homeworks will be evaluated on the basis of:

Accuracy The solution must be a reasonable and correct answer to the homework question.

Exposition The solution must clearly show the reasoning that was utilized to find it and the method of solution should be clearly apparent. Exegetic solutions will be ranked higher.

Beauty Good solutions will often require graphics and, with care, graphics can often beautifully explain the solution. The layout of the page, the quality of the supporting prose, the clarity of the graphics, and all that "je ne sais quoi" is fairly subjective but very important. The reader will include a judgment of your art in the ranking of homeworks.

Observation Supplemental observations provide aids in understanding and demonstrate mastery of a topic. An example of a supplemental observation might be something like, "Note that in the limit of long times, that the total concentration goes to zero. This is sensible because the boundary condition on mass flux is directed outward everywhere on the finite domain."

Laboratories will be graded on their completeness, demonstrated mastery of MATHEMATICA® for that assignment, and exposition.

Note that there will be times when you have two homework sets pending—this is done so that you can arrange your time conveniently.

Homework Calendar and Weighting

Not all homeworks are weighted equally:

Homework Schedule and Weighting

Homework	weight factor	Handed Out		Due Back	
		Lecture	Date	Lecture	Date
1	0.5	Lect. 1	7 Sept.	Lect. 5	16 Sept.
2	1.0	Lect. 4	14 Sept.	Lect. 7	23 Sept.
3	1.5	Lect. 6	21 Sept.	Lect. 8	3 Oct.
4	2.0	Lect. 10	07 Oct.	Lect. 15	21 Oct.
5	2.0	Lect. 21	14 Nov.	Lect. 23	28 Nov.
6	2.0	Lect. 23	28 Nov.	Lect. 27	7 Dec.

Note that there will be times when you have two homework sets pending—this is done so that you can arrange your time conveniently.

Late Policy

Students will be allowed to turn in one homework up to 3 days late, for the individual portion only. No second late homework will be allowed without formal documentation about an unforeseeable emergency. No late group homework portions will be accepted—no exceptions.

Laboratory assignments must be turned in during the laboratory period. You must show documentation of unforeseeable emergencies that prevent you from attending a laboratory period. Any missed laboratories must be made up by special arrangement. If for some reason, you cannot complete a laboratory during the laboratory period, you should send a paragraph explaining why you could not finish.

Textbook

We will use a fairly general textbook on applied mathematics (E. Kreyszig, *Advanced Engineering Mathematics*, Eighth ed., J.W. Wiley, ≈ 1200 pages). You'll notice that reading assignments do not follow the table of contents—while I like the book, there are pedagogical reasons for studying mathematics in the sequence we will follow in this subject. Extra material pertaining to materials science specifically will be created and placed on the web.

I have identified 66 sections of the book (330 pages in total) as required reading. The readings for each lecture will appear in the Lecture Notes. I hope you will keep up with the reading—I think it would be wise to give the material a cursory reading prior to the lecture and then read it more carefully before starting the homework.

This course is designated as a 12 (*3-1-8*) unit subject[1] Time spent awake at lectures and recitations is less than half of your job—reading and doing homework is the greater part.

Lecture Notes

Lecture notes (like these) will be available for you to print out for each lecture. The lecture notes will be available at: http://pruffle.mit.edu/3.016. These will supplement (not replace) the textbook. The lecture notes also serve as a guide to help the student understand what parts of the text are considered more relevant or important.

The specific purpose of the notes is to provide neatly typeset equations and graphics that will be used in the lecture along with a few observations. This will eliminate the time required

[1]Units at MIT are assigned under the following schema: *lec-lab-out* where *lec* is the number of lecture/recitation hours, *lab* is the number of laboratory hours, and *out* is the number of outside (reading, preparation, homework) hours per week. One MIT unit represents about 14 hours of semester work on the average.

to write and draw, perhaps a bit sloppily, for you in your notes and for me on the blackboard.

The lecture notes will have reading assignments printed at the beginning of each lecture; they will look like this:

Reading:
Kreyszig Sections: §6.1 (pp:304–09) , §6.2 (pp:312–18) , §6.3 (pp:321–23) , §6.4 (pp:331–36)

Much of what is important is spoken by the lecturer (or in the form of welcome questions and points of clarification by the students) and some explanatory notes will be written upon the blackboard. Mixing projected displays of equations and graphs with blackboard writing will allow incremental adjustments to the best learning pace.

The notes will have places for you to fill in auxiliary discussion and explanation. Those places will look like this:

You can use these notes in several ways. You could print them out before lecture and write your own lecture notes directly during the lecture. You could take lecture notes on your own paper and then neatly copy them onto a printout later. You could print them before lecture and write on them rapidly and then copy—neatly and thoughtfully—notes onto a freshly printed set of lecture. I recommend the latter for effective learning and the creation of a set of notes that might provide future reference material—but do whatever works for you.

The lecture notes will also refer to MATHEMATICA® notebooks available on the 3.016 website for downloading. These notebooks will be used as MATHEMATICA® sessions during the lectures to illustrate specific points and provide examples for you to help solve homework problems.

References to MATHEMATICA® notebooks look like this:

MATHEMATICA® Example: (notebook) Lecture-01

1. Starting Mathematica.

2. Defining symbols and performing simple operations on them.

3. Saving Mathematica and Quitting.

They will serve as place-holders in the lecture note when we switch from chalkboard and/or projected display of the notes to a live MATHEMATICA® session.

Lecture and Laboratory Calendar

This calendar will be updated throughout the semester: students should consult this calendar weekly to obtain the required reading assignments for the laboratory. It is also available in iCal format (subscribe to webcal://pruffle.mit.edu/dav/3016.ics; check off "subjects and notes" and "To Do items").

Week of 5–9 September

Lectures		
	Topics	**Reading**
M 09/05	Labor Day, No lecture	
W 09/07 Lect. 1	Course organization and introduction to Mathematica	Mathematica Notes *I*
F 09/09 Lect. 2	Introduction to Mathematica, assignment and evaluation, rules and replacement, procedural and functional programming	Mathematica Notes *II*
Laboratory		
F 09/09 Lab 1 (Not Graded)	Getting started with Mathematica	*Mathematica Help Browser* **Online Tutorial**

Week of 12-16 September

Lectures		
	Topics	**Reading**
M 09/12 Lect. 3	Mathematica graphics: basic plotting, data, two- and three-dimensional plotting, graphics primitives, formatting	Mathematica Notes *III*
W 09/14 Lect. 4	Mathematica: symbolic and numeric calculations, linear algebra, roots of equations	Mathematica Notes *IV*
F 09/16 Lect. 5	Mathematica: functional programming, packages, and file input/output	Mathematica Notes *V*
Laboratory		
F 09/16 Lab 2	Symbolic calculations and plotting	*Mathematica Help Browser* **Mathematica Book: sections 1.4.2, 1.7.1; Functions: Integrate, Simplify, NIntegrate, Plot, Plot3D, ContourPlot**

Week of 19-22 September

Lectures		
	Topics	**Reading**
M 09/19	MIT Holiday, No Lectures	
W 09/21 Lect. 6	Linear algebra: matrix operations, interpretations of matrix operations, multiplication, transposes, index notation	*Kreyszig* **6.1, 6.2, 6.3, 6.4** (pages: 304–309, 312–318, 321–323, 331–336)
F 09/23 Lect. 7	Linear algebra: solutions to linear systems of equations, determinants, matrix inverses, linear transformations and vector spaces	*Kreyszig* **6.5, 6.6, 6.7, 6.8** (pages: 338–341, 341–347, 350–357, 358–364)
Laboratory		
F 09/23 Lab 3	Solving linear systems of equations	*Mathematica Help Browser* **Mathematica Book Section 1.8.3, Functions: Inverse, Transpose, Eigensystem, matrix multiplication "."**

Week of 26-30 September

Laboratory Week: 3.016 does not meet.

Week of 3–7 Oct.

Lectures		
	Topics	**Reading**
M 10/03 Lect. 8	Complex numbers: complex plane, addition and multiplication, complex conjugates, polar form of complex numbers, powers and roots, exponentiation, hyperbolic and trigonometric forms	*Kreyszig* **12.1, 12.2, 12.6, 12.7** (pages: 652–656, 657–662, 679–682, 682–685)
W 10/05 Lect. 9	Matrix eigenvalues: eigenvalue/eigenvector definitions, invariants, principal directions and values, symmetric, skew-symmetric, and orthogonal systems, orthogonal transformations	*Kreyszig* **7.1, 7.2, 7.3** (pages: 371–375, 376–379, 381–384)
F 10/07 Lect. 10	Hermitian forms, similar matrices, eigenvalue basis, diagonal forms	*Kreyszig* **7.4, 7.5** (pages: 385–389, 392–396)

Laboratory		
F 10/07 Lab 4	File input/output, plotting data	*Mathematica Help Browser* **Mathematica Book 2.12.7, 2.12.8; Functions: Dimensions, Append. AppendTo, Do, Mean, StandardDeviation, ListPlot, Table, Graphics‘MultipleListPlot, Fit**

Week of 10–14 October

Lectures		
	Topics	**Reading**
M 10/10	Holiday, No lecture	
W 10/12 Lect. 11	Vector calculus: vector algebra, inner products, cross products, determinants as triple products, derivatives of vectors	*Kreyszig* **8.1, 8.2, 8.3, 8.4** (pages: 401–406, 408–413, 414–421, 423–427)
F 10/14 Lect. 12	Multi-variable calculus: curves and arc length, differentials of scalar functions of vector arguments, chain rules for several variables, change of variable and thermodynamic notation, gradients and directional derivatives	*Kreyszig* **8.5, 8.8, 8.9** (pages: 428–433, 444–446, 446–452)
Laboratory		
F 10/14 Lab 5	Statistics, fitting data, error analysis	*Mathematica Help Browser* **Mathematica Book: 3.8.2; Functions: Fit, FindFit; Package: Statistics‘NonlinearFit**

Week of 17–21 October

Lectures		
	Topics	**Reading**
M 10/17 Lect. 13	Vector differential operations: divergence and its interpretation, curl and its interpretation	*Kreyszig* **8.10, 8.11** (pages: 453–456, 457–459)
W 10/19 Lect. 14	Path integration: integral over a curve, change of variables, multidimensional integrals	*Kreyszig* **9.1, 9.2, 9.3** (pages: 464–470, 471–477, 478–484)
F 10/21 Lect. 15	Multidimensional forms of the Fundamental theorem of calculus: Green's theorem in the plane, surface representations and integrals	*Kreyszig* **9.4, 9.5, 9.6, 9.7** (pages: 485–490, 491–495, 496–505, 505-509)
Laboratory		
F 10/21 Lab 6	Graphical representations in three and higher dimensions	*Mathematica Help Browser* **Mathematica Book: 1.9.1—1.9.7, 1.9.9—1.9.11**

Week of 24–28 October

Laboratory Week: 3.016 does not meet.

Week of 31 October–4 November

Lectures		
	Topics	**Reading**
M 10/31 Lect 16	Multi-variable calculus: triple integrals and divergence theorem, applications and interpretation of the divergence theorem, Stokes' theorem.	*Kreyszig* **9.8, 9.9** (pages: 510–514, 515–520)
W 11/02 Lect. 17	Periodic functions: Fourier series, interpretation of Fourier coefficients, convergence, odd and even expansions	*Kreyszig* **10.1, 10.2, 10.3, 10.4** (pages: 527–528, 529–536, 537–540, 541–546)
F 11/04 Lect. 18	Fourier theory: complex form of Fourier series, Fourier integrals, Fourier cosine and sine transforms, the Fourier transforms	*Kreyszig* **10.5, 10.8, 10.9, 10.10** (pages: 547–549, 557–563, 564–568, 569-575)
Laboratory		
F 11/04 Lab 7	Review of Mathematica functions and graphics	*Mathematica Help Browser* **Mathematica Book: 1.9.1–1.9.9, 2.1.1, 2.2.1. 2.3.1, 2.4.1, 2.5.1, 2.6.1, 2.7.1**

Week of 7–11 November

Lectures		
	Topics	**Reading**
M 11/07 Lect 19	Ordinary differential equations: physical interpretations, geometrical interpretations, separable equations	*Kreyszig* **1.1, 1.2, 1.3** (pages: 2–8, 10–12, 14–18)
W 11/09 Lect. 20	ODEs: derivations for simple models, exact equations and integrating factors, the Bernoulli equation	*Kreyszig* **1.4, 1.5, 1.6** (pages: 19–22, 25–31, 33–38)
F 11/11	Veterans' Day, No lecture	

Week of 14–18 November

Lectures		
M 11/14 Lect. 21	Higher order differential equations: homogeneous second order, initial value problems, second order with constant coefficients, solution behavior	*Kreyszig* **2.1, 2.2, 2.3** (pages: 54–70, 72–75, 76–80)
W 11/16	3.014 Laboratory, no 3.016 lecture	
F 11/18	3.014 Laboratory, no 3.016 lecture	

Week of 21–25 November

Lectures		
	Topics	**Reading**
W 11/21	3.014 Laboratory, no 3.016 lecture	
W 11/23 Lect. 22	Differential operators, damped and forced harmonic oscillators, non-homogeneous equations	*Kreyszig* **2.4, 2.5, 2.8** (pages: 81–83, 83–89, 101–103)
F 11/25	MIT Holiday, no 3.016 lecture	

Week of 28 November–2 December

Lectures		
	Topics	**Reading**
M 11/28 Lect. 23	Resonance phenomena, higher order equations, beam theory	*Kreyszig* **2.11, 2.13 (beam theory only)** (pages: 111–116, 130–131)
W 11/30 Lect. 24	Systems of differential equations, linearization, stable points, classification of stable points	*Kreyszig* **3.1, 3.2** (pages: 152–157, 159–161)
F 12/02 Lect. 25	Linear differential equations: phase plane analysis and visualization	*Kreyszig* **3.3, 3.4** (pages: 161–169, 170–174)
Laboratory		
F 12/02 Lab 8	Solutions to ordinary differential equations	*Mathematica Help Browser* **Mathematica Book: 1.5.9, 3.5.11; Function: DSolve, NDSolve, NIntegrate**

Week of 5–9 December

Lectures		
	Topics	**Reading**
M 12/05 Lect. 26	Solutions to differential equations: Legendre's equation, orthogonality of Legendre polynomials, Bessel's equation and Bessel functions	*Kreyszig* **4.3, 4.5, 4.6** (pages: 111–116, 130–131, 228–232)
W 12/07 Lect. 27	Sturm-Louiville problems: eigenfunction, orthogonal functional series, eigenfunction expansions	*Kreyszig* **4.7, 4.8** (pages: 233–238, 240–248)
F 12/09	3.014 Laboratory continues, No more Maths lectures	

Week of 12–16 December

Laboratory Week: 3.016 does not meet.