Overview

This laboratory should give students experience with some of the basic methods for solving matrix equations and for finding the eigenvalues and eigenvectors of a matrix.

Tasks

Solving and Interpreting Systems of Linear Equations

The electrical conductivity, σ, is a second-rank tensor property that relates the current density vector, $\vec{J}$, to the electric field, $\vec{E}$ by

$$\vec{J} = \sigma \vec{E}$$

In a particular coordinate system, the electrical conductivity of a tetragonal tin single crystal was measured to be:

$$\sigma = \begin{pmatrix} 8.55 & 1.55 & 0 \\ 1.55 & 8.55 & 0 \\ 0 & 0 & 10.1 \end{pmatrix} \times 10^6 \quad (\text{ohm} \cdot \text{m})^{-1}$$

1. Find the current density vector and its magnitude when an electric field of 0.1 V/m is applied parallel to the "2" axis in this coordinate frame.
2. Find the electric field, $\vec{E}$, which will create a current density $\vec{J}$ of magnitude 10^{-6} coulomb $\text{s}^{-1}\,\text{m}^{-2}$ flowing *in the direction of* (111).
3. The vector
$$\hat{E} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$
represents electric fields with unit magnitude directed towards θ in the x–y plane. The angle, α, between two vectors $\vec{J}$ and $\vec{E}$ is given by
$$\cos\alpha = \frac{\vec{J} \cdot \vec{E}}{|\vec{E}|\,|\vec{J}|}$$
Plot the angle between the current density and the electric field for for all the unit vectors, $\hat{E}$, given above. In other words, plot $\alpha(\theta)$.
4. Find the eigenvalues and eigenvectors of tin's electrical conductivity.
5. Demonstrate that, if an electric field is applied in the direction of one of the eigenvectors calculated above, the current density will be parallel to the electric field.

Save your Work Save your work as a mathematica notebook: `3016_Lastname_Lab03.nb`.

Report

This homework will be graded. Your report on the work above should be ordered as it is above. Your report should include comments that would help one of your classmates understand what your work demonstrates. Send your report as a saved Mathematica notebook with name `3016_Lastname_Lab02.nb` to 3.016@pruffle.mit.edu.