Homework 2I — Solution to Individual Exercises

3.016 Mathematical Methods for Materials Scientists and Engineers
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```
Clear["Global`*"]
```

Individual Exercise I2-1: (Kreyszig MCG 6.2, p. 77)

Using the matrices provided

```
A = {{5, -3, 0}, {6, 1, -4}}
```

```
{{5, -3, 0}, {6, 1, -4}}
```

```
% // MatrixForm
```

$$\begin{pmatrix} 5 & -3 & 0 \\ 6 & 1 & -4 \end{pmatrix}$$

```
B = {{-2, 4, -1}, {1, 1, 5}}
```

{{−2, 4, −1}, {1, 1, 5}}

```
% // MatrixForm
```

$$\begin{pmatrix} -2 & 4 & -1 \\ 1 & 1 & 5 \end{pmatrix}$$

From here on it is pretty much plug and chug:

```
A.Transpose[A] // MatrixForm
```

$$\begin{pmatrix} 34 & 27 \\ 27 & 53 \end{pmatrix}$$

```
Transpose[ A].A  // MatrixForm
```

$$\begin{pmatrix} 61 & -9 & -24 \\ -9 & 10 & -4 \\ -24 & -4 & 16 \end{pmatrix}$$

```
(Transpose[ A].A ).(Transpose[ A].A ) // MatrixForm
```

$$\begin{pmatrix} 4378 & -543 & -1812 \\ -543 & 197 & 112 \\ -1812 & 112 & 848 \end{pmatrix}$$

```
(A + B).Transpose[A - B] // MatrixForm
```

$$\begin{pmatrix} 13 & 24 \\ 36 & 26 \end{pmatrix}$$

A-Individual Exercise II-2: (Kreyszig MCG 6.10, p. 77)

Define the matrix with general coefficients by

AMatrix $= \{\{a_{11}, a_{12}\}, \{a_{21}, a_{22}\}\}$

$\{\{a_{11}, a_{12}\}, \{a_{21}, a_{22}\}\}$

Remind ourselves what the determinant will be:

```
Det[AMatrix]
```

$-a_{12}\, a_{21} + a_{11}\, a_{22}$

Now examine the inverse

```
Inverse[AMatrix]
```

$$\left\{\left\{\frac{a_{22}}{-a_{12}\,a_{21}+a_{11}\,a_{22}},\,-\frac{a_{12}}{-a_{12}\,a_{21}+a_{11}\,a_{22}}\right\},\left\{-\frac{a_{21}}{-a_{12}\,a_{21}+a_{11}\,a_{22}},\,\frac{a_{11}}{-a_{12}\,a_{21}+a_{11}\,a_{22}}\right\}\right\}$$

and use Replace to simplify the expression for Det[AMatrix] in the denominators. Viewing in MatrixForm we have the relations in the desired form:

```
% /. Det[AMatrix] → DetA // MatrixForm
```

$$\begin{pmatrix} \frac{a_{22}}{\text{DetA}} & -\frac{a_{12}}{\text{DetA}} \\ -\frac{a_{21}}{\text{DetA}} & \frac{a_{11}}{\text{DetA}} \end{pmatrix}$$

A-Individual Exercise II-3: (Kreyszig MCG 6.12, p. 78)

```
Clear[A, B]
```

Define the matrices A and B by

```
A = {{0, −2, −1}, {−2, 3, 2}, {−1, 2, 1}};
B = {{1, 2, 3}, {2, 3, 4}, {3, 4, 6}};
```

Define the column vectors x and w by

```
x = {{x1}, {x2}, {x3}};
w = {{w1}, {w2}, {w3}};
```

Now calculate y from w by a two-step method of successive matrix multiplications involving the intermediate vector x

```
x = B.w
```

```
{{w1 + 2 w2 + 3 w3},
  {2 w1 + 3 w2 + 4 w3}, {3 w1 + 4 w2 + 6 w3}}
```

```
firsty = A.x
```

```
{{-3 w1 - 4 w2 - 6 w3 - 2 (2 w1 + 3 w2 + 4 w3)},
  {-2 (w1 + 2 w2 + 3 w3) +
      3 (2 w1 + 3 w2 + 4 w3) + 2 (3 w1 + 4 w2 + 6 w3)},
  {2 w1 + 2 w2 + 3 w3 + 2 (2 w1 + 3 w2 + 4 w3)}}
```

Now use the matrix product A.B to compute y directly from w

```
CMat = A.B
```

```
{{-7, -10, -14}, {10, 13, 18}, {6, 8, 11}}
```

```
secondy = CMat.w
```

```
{{-7 w1 - 10 w2 - 14 w3},
  {10 w1 + 13 w2 + 18 w3}, {6 w1 + 8 w2 + 11 w3}}
```

Check to ensure that the two results are the same (FullSimplify is used to ensure that the expressions have exactly the same form)

```
FullSimplify[firsty] == FullSimplify[secondy]
```

```
True
```

Here is the inverse of CMat:

```
InvCMat = Inverse[CMat]
```

```
{{1, 2, -2}, {2, -7, 14}, {-2, 4, -9}}
```