

Solution to Sample Problems in Recitation 2

Problem 1

Calculate the amount of work which is done when 1 mol of water is vaporized at 100 C and 1 atm pressure. Assume that water vapor behaves as an ideal gas. Clearly state any assumptions necessary to solve the problem. Define the system and its constraints.

Solution:

Sometimes it is useful to draw a diagram representing the system to study:

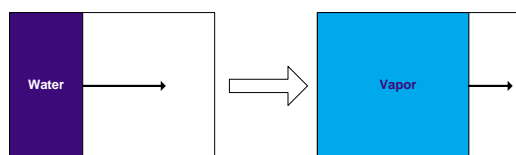


Figure 1: Vaporization of Water at 100C and 1 atm

As we can see from the diagram, the volume occupied by the water vapor is much greater than that occupied by water. We will verify this through calculations.

In order to properly analyze this thermodynamic system it is important to state clearly the assumptions made. In Thermodynamics, we always have to be very precise regarding the constraints and characteristics of the system, otherwise, we might end up with the wrong conclusions and sometimes a not-so-good grade :-).

For this system, we can make the following assumptions:

- The right boundary of the system is free to move back and forth.
- This movement occurs without friction, so to calculate the total work done by the system we will not have to consider any friction losses.
- The system is closed, so there is no mass transfer across its boundaries.

- Since the system is unconstrained in that wall, we must assume that there is no pressure buildup in the container(isobaric process).

We can also assume that the only way the system can exert work on its surroundings is through its expansion and therefore, the following expression may be used:

$$W = \int Pdv = P(\bar{V}_g - \bar{V}_l) \quad (1)$$

Where \bar{V} corresponds to the molar volume (we are asked to obtain the amount of work per mol).

Now, we are told that we can consider water vapor as an ideal gas (under normal conditions this is a very good approximation) and therefore we use our old friend, the ideal gas equation of state:

$$P\bar{V} = RT \quad (2)$$

From this equation of state it is possible to get the molar volume of water vapor at a given pressure and temperature and therefore the only remaining unknown is the molar volume of liquid water.

Water has a density of $1gcm^{-3}$. Since the molecular weight of water is $18gmol^{-1}$, the molar volume of liquid water under normal conditions is approximately $18E - 6m^3mol^{-1}$.

Finally, the expression for the work of vaporization for our particular process is:

$$W = P\left(\frac{RT}{P} - 18E - 6\right) \quad (3)$$

or just

$$W \approx RT \quad (4)$$

since the molar volume of water vapor is about 1700 times greater than the molar volume of water.

Problem 2

7.5 kJ of heat are added to a *closed* system while its internal energy decreases by 12kJ. How much energy is transferred as work? For a process causing the same change of state but for which the work is zero, how much heat is transferred?

Solution:

This problem is really straightforward, provided a proper sign convention is defined for the heat and work absorbed and released by the system.

In this particular case we can consider that heat and work absorbed by the system are positive quantities (You can pick whatever sign conventions suits you better, but you must be consistent).

Now, for the first case, we have

$$\Delta U = Q + W \rightarrow -12kJ = 7.5kJ - 19.5kJ \quad (5)$$

This means that the system absorbed heat and at the same time it exerted work on its environment.

When no work is added to the system, the total change in internal energy must be due to a heat transfer process:

$$\Delta U = Q + 0 \rightarrow -12kJ = -12.5kJ - 0kJ \quad (6)$$

Thus, the decrease in internal energy of the system must be due to a release of energy in the form of heat.

Problem 3

This problem has the purpose of illustrating the equivalence between heat and work:

A perfectly insulating and sealed container is filled with 100 kg of water at 20C. A stirrer is installed inside the water container, and is made to turn by gravity, using a weight of mass of 100 kg (outside the container). The weight falls *very very* slowly through a distance of 1 m in driving the stirrer. Since the mechanism was designed by some bright MIT kids, the efficiency of the stirring mechanism is such that all the work done by the weight is transferred

to the water. Assume also that the acceleration of gravity is 10ms^{-2} .

- State all assumptions concerning the system, with constraints included
- Calculate
 1. The amount of work done on the water
 2. The internal-energy change of the water
 3. The final temperature of the water (Cp of water: $4.186\text{kJkg}^{-1}\text{C}^{-1}$)
 4. Amount of heat that must be removed to return the water to its initial temperature
 5. The total energy change of the universe because of the processes of lowering the weight and of cooling the water

As usual, we begin by figuring out what the system might look like:

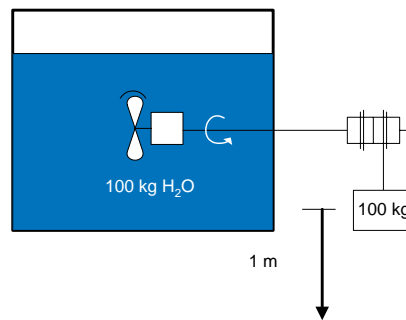


Figure 2: Heating up water by doing work on it

As was said above, it is always useful to state the assumptions and constraints so we know exactly what the system is "allowed" to do:

- Since the system is surrounded by insulating walls, there is no heat transfer from or towards the system.
- No mass is lost or gained by the system during the process.
- Since the weight falls very slowly, we can assume that the system remains at equilibrium at all times. There is no gradient in temperature across the water tank.

- There are no friction losses, so work is transmitted 100% efficiently from the falling weight to the water tank.
- We can assume that initially, the water in the container and its surroundings have the same temperature.
- Pressure within the container remains constant. This means we can use C_p .

Now we can proceed to solve the problem:

1. Assuming negligible friction losses during the process of lowering the weight, the work performed on the water corresponds with the decrease in potential energy of the block:

$$W = m_{block}g\Delta h \quad (7)$$

2. *In an adiabatic system, the change in its internal energy is only due to the work transferred to or from the system, since there is no heat transfer involved ($\Delta U = 0 + W$).* Therefore:

$$\Delta U_{Water} = m_{block}g\Delta h \quad (8)$$

3. Energy is transferred from the weight to the water through mechanical work. In the following lectures you will learn that the specific heat (in this case C_p) represents the ability of any given material to absorb thermal energy. Using freshman physics concepts, we can represent the change in internal energy of the water as:

$$\Delta U = m_W C_p \Delta T \quad (9)$$

The change in temperature ΔT is:

$$\Delta T = \frac{m_{block}g\Delta h}{m_W C_p} \quad (10)$$

4. If later we remove the insulation, then it is possible to have heat transferred from the *now* non-adiabatic system to its surroundings. Since

$T_{final} > 20C$, the Zeroeth Law tells us that heat will flow from the water to its surroundings. Since no work is involved, the heat transferred to the surroundings must come from the internal energy accumulated by the water:

$$Q = -m_W C_p \Delta T \quad (11)$$

5. *Since the universe is a perfect isolated system (with no work or heat transferred to or from it), the total change in energy of the system is zero:*

$$\Delta U_{isolated} = 0 \quad (12)$$