1. A state function for a Van der Waals gas is given by an equation between thermodynamic variables that depend on model parameters A,B, and a physical constant R:

$$(P + \frac{AN^2}{V^2})(V - NB) \quad = \quad NRT$$

where AN^2/V^2 is referred to as the internal pressure due to the attraction between molecules and NB is an extra volume, sometimes associated with the volume per molecule.

Write out a differential expression for dN in terms of differentials of the thermodynamic variables.

The solution is pretty straightforward. One way is to differentiate the entire expression and group the terms corresponding to dN, dP, dT, and dV. Another way to do is by implicit differentiation. The real gas equation can be rewritten such that,

$$\begin{aligned} N &= N(T, V, P) \\ dN &= \left(\frac{\partial N}{\partial P}\right)_{T,V} dP + \left(\frac{\partial N}{\partial V}\right)_{T,P} dV + \left(\frac{\partial N}{\partial T}\right)_{P,V} dT \end{aligned}$$

In an equivalent way, you could have written the function $P = P(V, T, N)$ and extract dN from the following.

$$dP \quad = \quad \left(\frac{\partial P}{\partial N}\right)_{T,V} dN + \left(\frac{\partial P}{\partial V}\right)_{T,N} dV + \left(\frac{\partial P}{\partial T}\right)_{N,V} dT$$

For instance, the first term $(\frac{\partial P}{\partial N})_{T,V}$ can be evaluated as

$$\left(\frac{\partial P}{\partial T}\right)_{P,V} = \frac{NR}{V - NB}$$

Using any one of the methods you would get

$$dN \quad = \quad \frac{(V - BN)dP + (P - (AN^2/V^2) + (2ABN^3/V^3))dV - RNdT}{BP + RT + (3ABN^2/V^2) - (2AN/V)}$$

2. Each day a certain amount of water evaporates from the oceans, lakes, and earth surface and forms water vapor and clouds in the atmosphere. Each day a certain amount of rain falls back to the earth. Make the reasonable assumption that, on the average, the energy consumed by evaporating and liftin the water is equal to the energy produced by condensation and rain falling back to earth. To evaporate one mole of water, approximately 41090 joules of heat

are required and an equivalent amount is expelled when a mole of water condenses. Estimate the amount of work required each day to produce the rain that raineth. That is, the work to evaporate and lift the water. You may need to find data to help make this estimate such as the average height of a rain cloud, state what those data are and from whence they came. Discuss what is supplying this daily energy. Compare this daily energy consumption with the energy produced each year in metropolitan Boston.

From the problem statement,

$$\text{The energy to lift water} = \text{energy produced by condensation and rain}$$

The first step would be to find the volume of water evaporated in the surface of earth. Since all the water is assumed to precipitate and eturned as rain, an average rainfall data and earth surface area would be needed to estimate this. A simplifying assumption for the calculation of volume of water precipitated could be that the curvature of surface is very small compared to the average height of precipitation. (The following data are from problem solutions by the group Ryan Clark, Daniel MacPhee, Taylor Schildgen, Alex Bradley - reported taken from climate prediction center, 1979-2000 data).

global precipitation average per day = 3mm

radius of earth = 6371 kms

surface area of earth = $5.1 * 10^{14}\text{m}^2$

volume of precipitation = surface area * average precipitation = $3.0*10^{-3} * 5.1 * 10^{14}$ m^3 = $15.3*10^{12}$ m^3

mass of precipitation = density * volume = $15.3*10^{15}$ kg (density of water = 1gm/cc)

number of moles of water = total mass / molar mass = $15.3/18 * 10^{15} = 8.5 * 10^{14}$

Next step is to calculate the energy spent on lifting this estimated volume of water to the atmosphere. The energy needed to evaporate water is given in the problem statement as 41090 joules per gram. you would also need some information on the average height these drops are lifted or the average cloud height to calculate its potential energy change. The total energy per day day can be estimated from these calculations. This amount of energy is consumed everyday by the water that raineth.

A reasonable distance at which clouds are formed could be around 5000 kms.

The energy needed to lift the mass of water to this height = m*g*h = $15.3 * 10^{15} * 9.8 * 5000$ Joules = $7.497*10^{20}$ joules

The energy needed to evaporate the entire volume of water = number of moles * energy for evaporation per mole = $8.5 * 10^{14} * 41090$ Joules = $3.49265 * 10^{19}$ Joules.

The total energy to evaporate and lift = $(7.497 + .85)*10^{20}$ Joules = $8.347*10^{20}$ Joules.

Discussion on what is supplying the energy to lift this amount of water can be on fraction of sun's energy reaching earth, on the fraction of water surface available, average global temperature, ozone layers and anything else that might affect the supply of energy to evaporate and lift water vaopur.

Energy produced each year in metropoliton boston can be estimated from the average energy consumption of a Bostonian and the population of boston. Now this can be compared with the calculated total energy consumption to make rain that rains per day.

The per capita consumption of energy in US[1] (for 1999) = 13451 kWh

Population of Boston metropoliton = 5,815,331.

Energy consumed in Boston per day = 13541 * 10^3 * 5815331 * 60 / 365 Joules = 1.294 * 10^{13} Joules

Assuming that same amount of energy is being produced to feed the consumption, the ratio of energy produced in boston to the energy to make rain that rain-eth is

Energy produced in Boston / Energy needed to make rain = 1.294 * 10^{13} joules / 8.34 * 10^{20} joules = 1.54 * 10^{-8}

[1]http://www.iea.org/stats/files/selstats/keyindic/country/us.htm