Massachusetts Institute of Technology

Kinetic Processes in Materials

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Problem Set 7: Due Wed. April 17, 2002, Before 5PM in 4-049

Exercise 7.1

Use the symmetric molar regular free energy of mixing for a binary alloy of A and B at fixed pressure.

$$\begin{aligned} \overline{\Delta G^{\mathcal{RS}}} &= \overline{\Delta H^{\mathcal{RS}}} - T\overline{\Delta S^{\mathcal{IS}}} \\ &= X_A X_B \omega^{\mathcal{RS}} - T[(-R)(X_A \log X_A + X_B \log X_B)] \end{aligned} \tag{1}$$

and do the following.

7-1-i Determine the critical temperature T_c in terms of $\Omega^{\mathcal{RS}}$ and R.

7-1-ii Plot the equilibrium compositions from $T = 0$ to $T = 1.1T_c$.

7-1-iii Plot $\mu_A(X_B)$ for $0 \leq X_B \leq 1$.

7-1-iv Plot $\mu_A(X_B)$ versus $\mu_B(X_B)$ for $0 \leq X_B \leq 1$.

Exercise 7.2

Consider the stagnation problem associated with the disappearance of a nearly cylindrical grain in a thin sheet with thickness h.

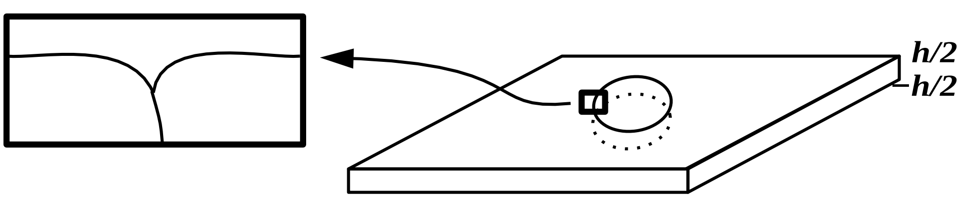

Figure 7-2-i: Illustration of disappearing grain in a thin sheet. The circular boundary groove, radius R_g, which forms on each surface creates a pinning force resisting boundary motion.

If a groove develops as shown, the grain boundary can become "pinned."

7-2-i Show that, for a pinned boundary, $r(z) = R_w \cosh(z/R_w)$ is the equilibrium shape of the grain boundary if all interfaces are isotropic.

7-2-ii Calculate the net force on the groove due to the grain when the radius of the groove $R_g = h$. Note that $\alpha\beta = \cosh(\beta/2)$ has two solutions when $\alpha = 1$, $\beta = 1.1787$ and $\beta = 4.2536$.

7-2-iii $\alpha\beta = \cosh(\beta/2)$ ceases to have any solutions when $\alpha < 0.75$. What happens to the grain when R_g deceases to about 3/4 h?

Exercise 7.3

Calculating the fastest growing and smallest unstable wavelengths for a cylinder which is evolving due to surface diffusion.

Start with a uniform cylinder and perturb with an infinitessimal pertubation $R(z,t) = R_o + \epsilon(t)\sin 2\pi z/\lambda$. Use the small slope approximation for the surface diffusion equation:

$$\frac{\partial R}{\partial t} = D_S^\star \frac{\partial^2 \kappa}{\partial z^2}$$

Find an expression for $\epsilon(t)$ and maximize with respect to λ.

Exercise 7.4

Determine the fastest growing and smallest unstable wavelengths (if they exist) for:

7-4-i a nonconserved order parameter, $\eta(x)$ with homogeneous free energy density:

$$f(\eta) = f_s((1+\eta)(1-\eta))^4$$

7-4-ii a conserved order parameter, $c(x)$ with homogeneous free energy density:

$$f(c) = \frac{2^8 f_s}{c_\beta - c_\alpha)^8}((c-c_\alpha)(c-c_\beta))^4$$