MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Kinetic Processes in Materials

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Problem Set 3: Due Mon. March 4, 2002, Before 5PM in 4-049

Exercise 3.1

Please solve Excercise 5.2 in *KPIM*.

Exercise 3.2

Please solve Excercise 5.5 in *KPIM*.

Exercise 3.3

Please solve the one-dimensional diffusion equation on the infinite domain for the following initial and boundary conditions:

$$c(x,t=0) = \begin{cases} c_o\frac{x}{a} & 0 < x < a \\ c_o(2-\frac{x}{a}) & a < x < 2a \end{cases}$$

$$\frac{\partial c}{\partial x}(x=0,t) = 0 \qquad \frac{\partial c}{\partial x}(x=\infty,t) = 0$$

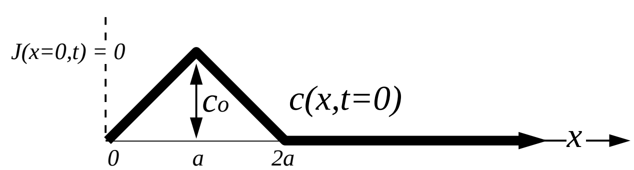

Figure 3-3-i: Trianglular initial conditions on the semi-infinite domain with zero-flux condtions at origin.

Part 1 Please find the time-dependent solution to this problem by using the superposition method.

Part 2 Please show how you would find the same solution using the Laplace transform. Find the solutions, $\hat{c}(x,p)$ which would need to be back-transformed to find the solution, but don't bother performing the back-transformation.

Hints

Use the zero-flux plane at $x = 0$ as a symmetry plane.

Note that the Laplace transform be continuous at $x = a$ and that the flux must be continuous.

Don't try to back-transform, but note that the following properties of the Laplace transform may be useful if used in combination with the table from Crank or the one from this year's lecture notes page 67:

$\hat{c}(p,x)$	$c(x,t)$
$\hat{c}(x,p/a)$	$ac(x,at)$
$\hat{c}(x,p-a)$	$e^{at}c(x,t)$
$e^{-ap}\hat{c}(p,x)$	$\mathrm{H}(x,t-a) = \begin{cases} c(x,t-a) & t > a \\ 0 & t < a \end{cases}$

Exercise 3.4

Please solve the diffusion equation to find a solution to the following problem and discuss whether your solution makes physical sense.

Consider a one cubic meter spherical initial source of argon at STP embedded in an infinite space of nitrogen, also at STP. Suppose the source is centered at the origin at time zero. Using reasonable values for the diffusivity of argon, calculate the time required for the number of argon atoms *within* a sphere of radius 3×10^8 meters, also centered at the origin, to decrease by exactly one.

Exercise 3.5

Please solve the one-dimensional diffusion equation on the finite domain $-L/2 \leq x \leq L/2$ for the following initial and boundary conditions:

$$c(x,t=0) = \begin{cases} \frac{c_0}{2} & \frac{-3L}{8} \leq x \leq \frac{-L}{8} \\ 0 & \frac{-L}{8} < x < \frac{L}{8} \\ c_0 & \frac{L}{8} \leq x \leq \frac{3L}{8} \end{cases}$$

$$J(x=-L/2) = 0 \qquad J(x=L/2) = 0$$

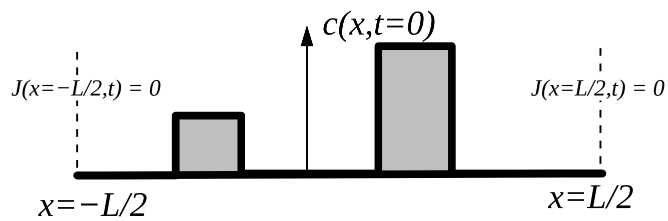

Figure 3-5-ii: Two square step initial contions for a finite domain with zero-flux condtions at edges.

Part 1 Please find an expression for the time dependence of the flux at $x = 0$.

Part 2 Please show that the cumulative flux across the plane $x = 0$ is consistent with the value that is apparent from the steady-state solution for this problem.